# Geometry

*End of Course Success*

*Complete Student Lessons*

TEXAS EDUCATION AGENCY

# Triangular Thinking

# Geometry EOC Success
## Lesson Plan Summary: *Triangular Thinking*

**Topic:** Using the constructions of midpoints of the sides of triangles to form conjectures

**CCRS:** In this lesson, the student will:

- make and validate geometric conjectures.
- develop and evaluate convincing arguments.
- use various types of reasoning.
- use mathematics as a language for reasoning, problem solving, making connections, and generalizing.

| | |
|---|---|
| **Content Objective:** The student uses geometric constructions to make, test, and justify conjectures. | **Language Objective:** C3(C) The student is expected to learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions. |
| **Vocabulary:** midpoint, congruent, similar | **Prior Knowledge:** Students are expected to be familiar with the triangle congruency relationships and proving techniques. |

| RtI Tier I Differentiation Activity | Instructional Phase | Enrichment Differentiation Activity |
|---|---|---|
| *** Mini-teach**: Similarity is first introduced in 7th grade and congruency in 5th grade. Explicit instruction* of these concepts will facilitate students' understanding of the triangular midpoint theorem.<br><br>**Engage:**<br>Students having difficulty with vocabulary will develop a Frayer model small group poster.<br><br>**Explore:**<br>Groups may be assigned based on student level to allow more directed guidance where needed using a selection of the activities provided below. | **Engage: Journal assignment**<br>Students will be asked to complete the three sentence stems on ***Engage: Triangular Thinking Sentence Stems*** handout. Additional Materials: Student Journals<br><br>**Explore: *Triangular Thinking***<br>Students will be provided with at least one of the triangles from ***Triangular Thinking 1 Cut-Outs***. If students are working in groups, each student in the group will have a different triangle than his/her group mates. ***Triangular Thinking 1 Student Instructions*** may be projected or printed for students.<br><br>**Explain: *Triangular Thinking***<br>Students will share their conjectures from the Triangular Thinking 1 activity with a classmate and provide support for why they think their conjectures are true using | • Students who are ready for enrichment will choose a method of proof to complete the ***Triangular Thinking 4*** activity.<br><br>• Students will be provided with a conjecture that must be tested and shown to be false via a counterexample on ***Triangular Thinking 5*** activity |

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<ul>
<li>Triangular Thinking 3 Composing Triangles: Using congruent triangles (provided in Triangular Thinking 3 Composing Triangles Cut-Outs) students will construct a larger triangle. After students' are guided to see similarity of small to large and see side measure relationships, they will build a new triangle in the student journal.</li>
<li>Students demonstrating a need for assistance in developing a conjecture will be encouraged to utilize concrete tools such as Patty Paper™, ruler, protractor, etc.</li>
<li>Triangular Thinking 2 Card Match: Using a pictorial demonstration of thinking process for deriving a conjecture, students will be asked to match pictures (provided in Triangular Thinking 2 Card Match Cut-Outs) to verbal steps.</li>
</ul>
*Explicit Instruction includes teaching components such as:
<ul>
<li>clear modeling of the solution specific to the problem;</li>
<li>thinking the specific steps aloud during modeling;</li>
<li>presenting multiple examples of the problem and applying the solution to the problems; and</li>
<li>providing immediate corrective feedback to the students on their accuracy.</li>
</ul>
MSTAR I Presenter's Guide pg 173 (2009)
</td>
<td>geometric/hands-on explanations as well as axiomatic explanations.</td>
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<td>
Formative Assessment: Triangular Thinking<br>
Each student will receive a copy of Closing the Loop: Triangular Thinking. Students will be given an imaginary group's assignment and conjecture. From this information, each student will determine whether the group's conjecture is true and provide justification for their answer.<br><br>
Note:<br>
Formative assessment items test concepts taught in the lesson and provide teachers valid information on whether students learned the concepts, principles and skills related to the lesson.<br>
A transfer assessment question provides information on whether the students can take the concepts from the lesson and apply them in a novel situation.<br><br>
Triangular Thinking 5 could be used as a transfer assessment for this lesson. Many times state assessments require transfer of knowledge, therefore; both types of questions should be used. It is necessary to remember transfer items require students have a wide range of examples; these provide the background knowledge essential for transfer of information.
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**In your journal, complete the following sentence stems:**

1. **I can find the midpoint of a segment by …**

2. **I can tell if two triangles are congruent by …**

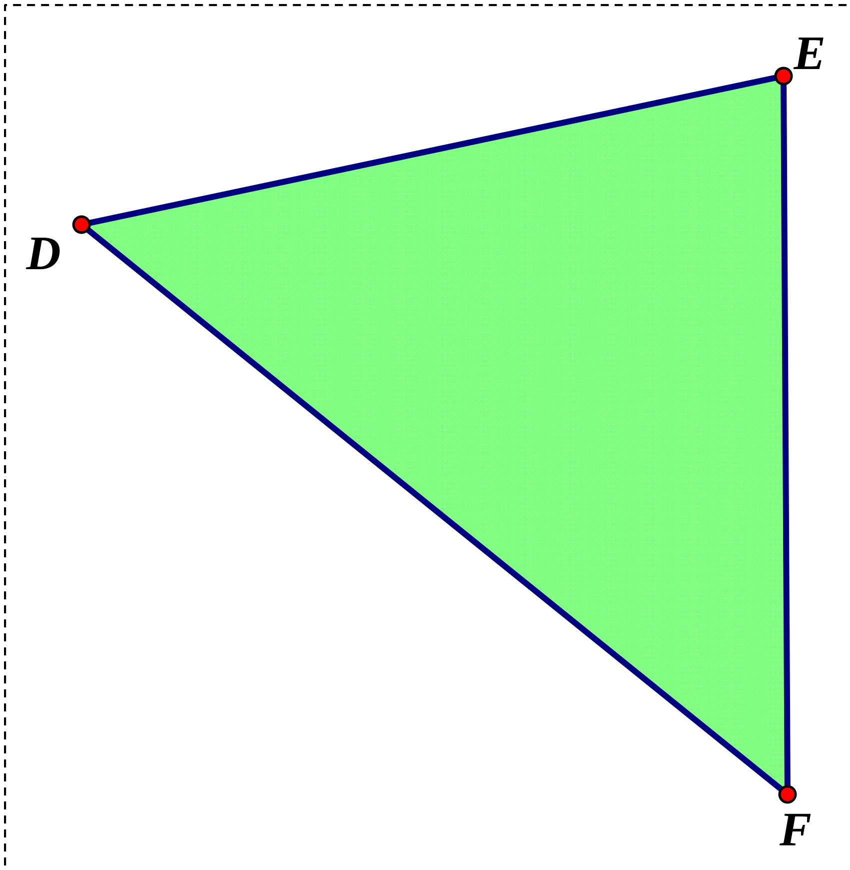
3. **I can tell if two geometric figures are similar by …**

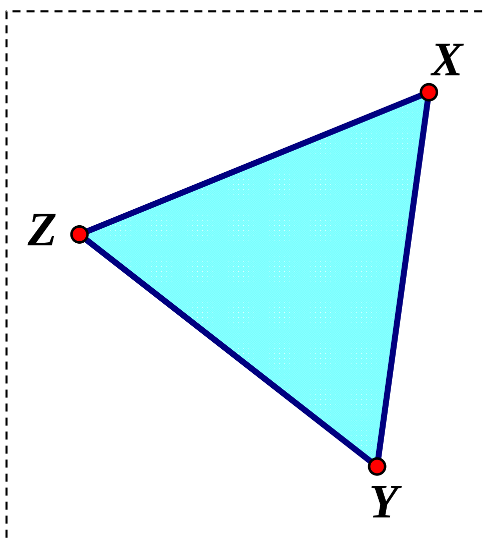
Student instructions:

- Construct the midpoint of each of the sides of the triangle.
- Connect the midpoints of the sides with line segments.
- Notice that four small triangles have now been constructed within the original triangle.
- Make conjectures about the triangles.
- Explain to a classmate why you believe that your conjectures are true.

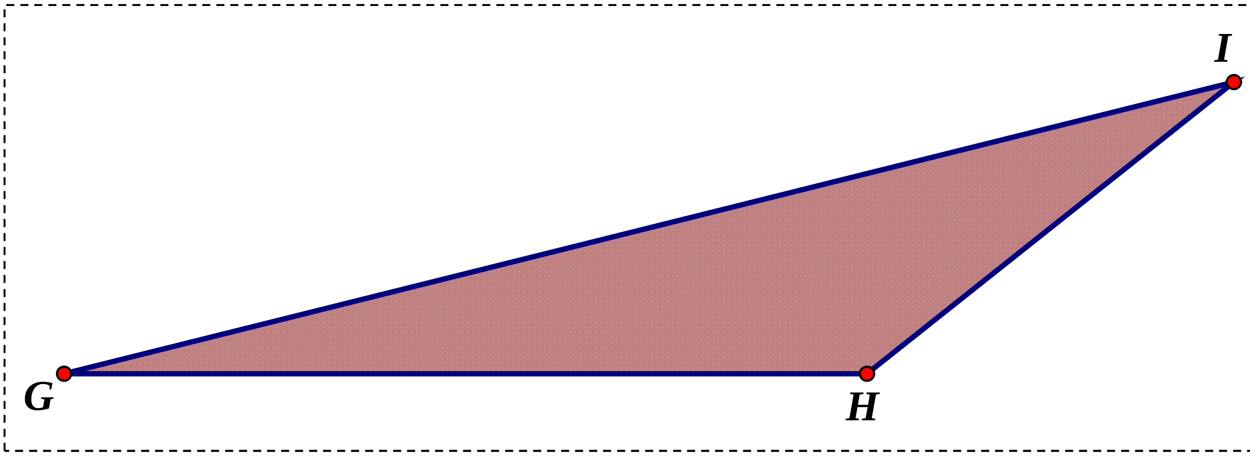
# Triangular Thinking 1 Cut-Outs

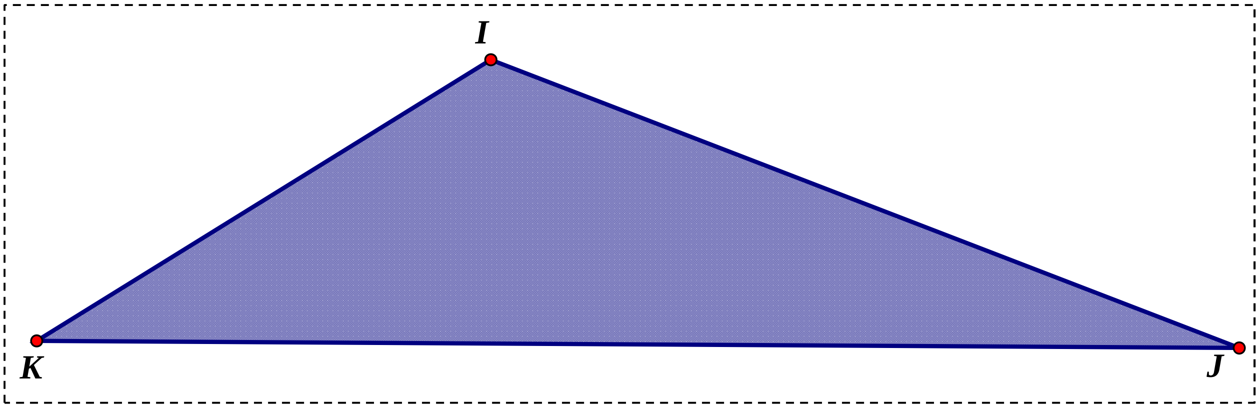
**Cut out triangles along dotted lines. Each student will need at least one. Make sure each student in each group has a different triangle. There are 8 triangles provided.**

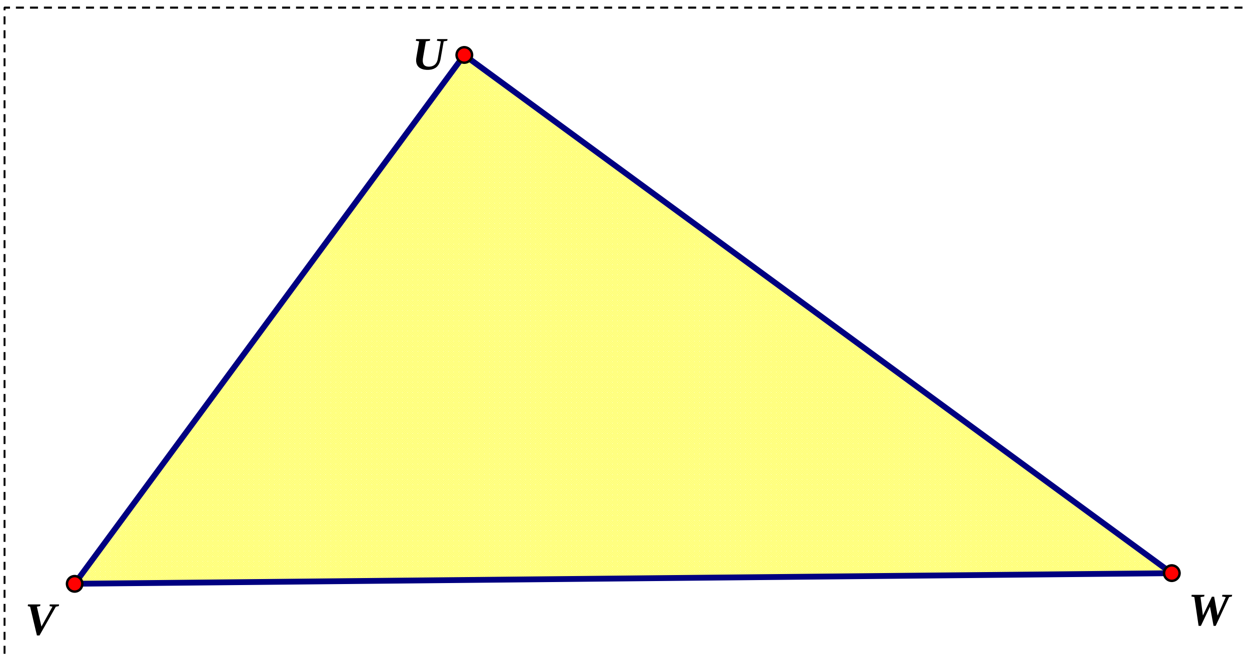


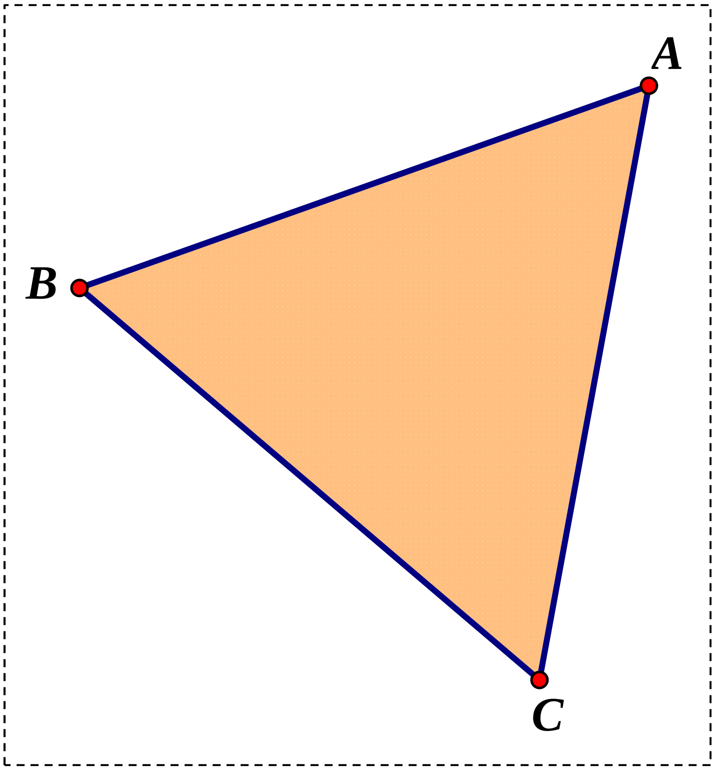
# Triangular Thinking 1 Cut-Outs

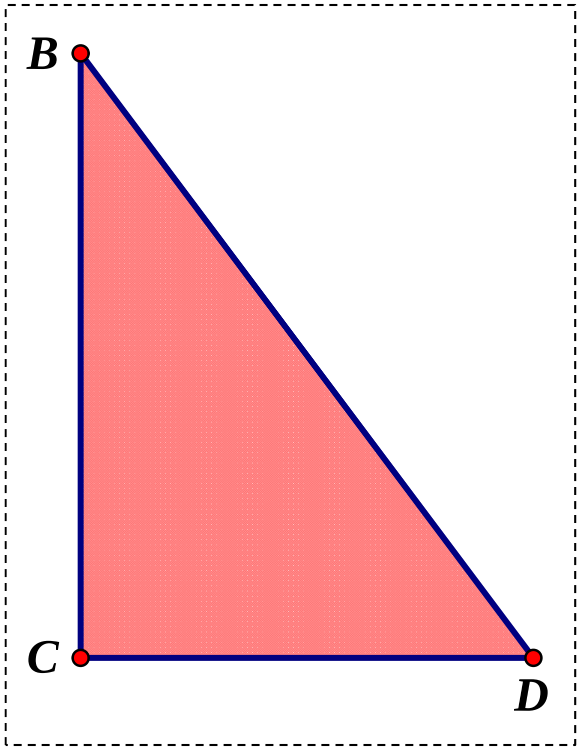


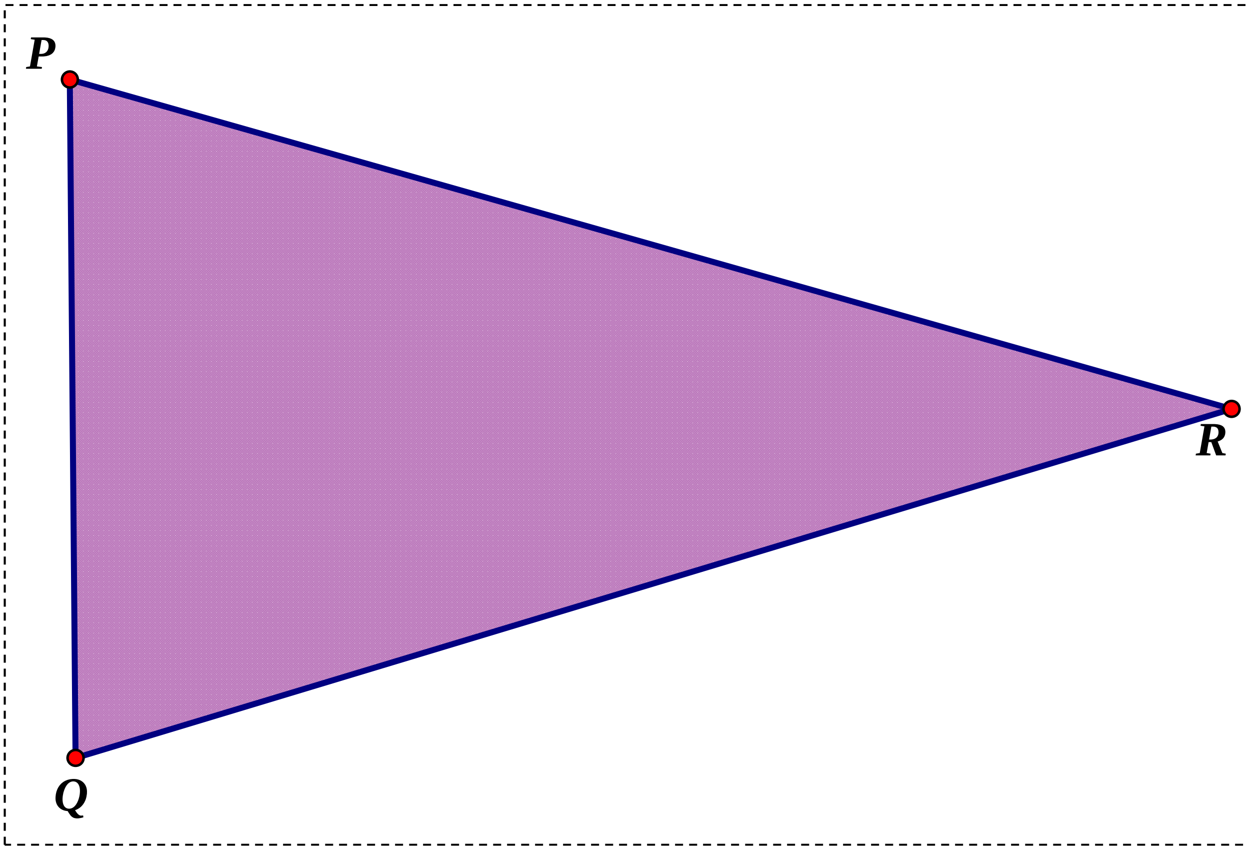



## Triangular Thinking 1 Cut-Outs



# Closing the Loop: Triangular Thinking

Students in another geometry class were asked to complete the following steps:

1. Draw any triangle.
2. Choose two sides of the triangle and find the midpoints of those sides.
3. Connect the midpoints with a line segment.
4. Examine your drawing and make a conjecture.

The students determined the following conjecture:

***The line segment that connects the midpoints of two sides of a triangle is half the length of the side opposing the angle formed by those sides.***

Do you agree with this conjecture? Provide justification for your answer.

# Triangular Thinking 2 Card Match

Daxton, one of your classmates, performed the following steps leading him to make the two conjectures provided within the steps below. Unfortunately, another classmate bumped into the desk and all the pictures that Daxton created fell on the floor. Help Daxton reorder his pictures by matching each picture to the statement that describes it. Fill in the blank on the picture with the correct step number so Daxton won't have this problem again.

***Daxton's steps:***

1. Given $\triangle ABC$.
2. Construct the midpoint of each of the sides of $\triangle ABC$.
3. Connect the midpoints with line segments and notice that there are four small triangles inside the original.
4. Cut out the four small triangles and see that the corresponding sides of each are congruent.

***<u>Daxton's Conjecture 1</u>: Given any triangle ABC. If the midpoints of the sides of that triangle are connected, the four small triangles that result are congruent.***

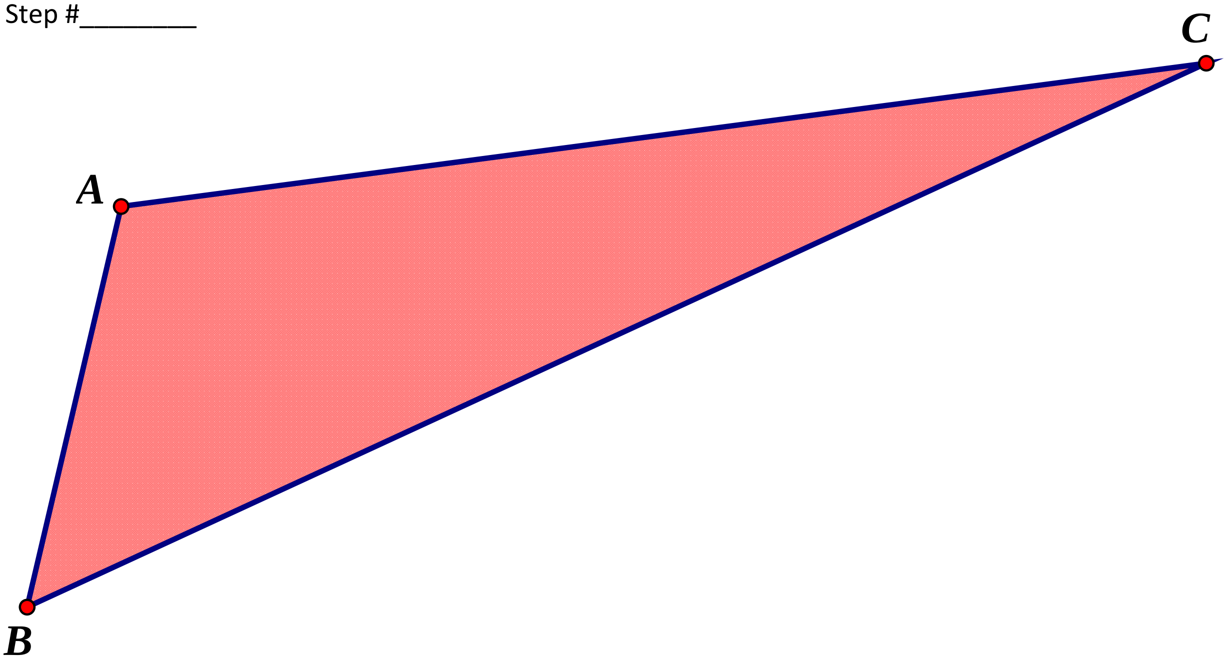
5. Comparing the side lengths of $\triangle ABC$ with the side lengths of $\triangle DEF$ (the triangle formed from connecting the midpoints), shows that there is a common ratio of 2 between all corresponding sides.

***<u>Daxton's Conjecture 2</u>: Given any triangle ABC. The triangle formed by connecting the midpoints of the original triangle is similar to the original ABC.***

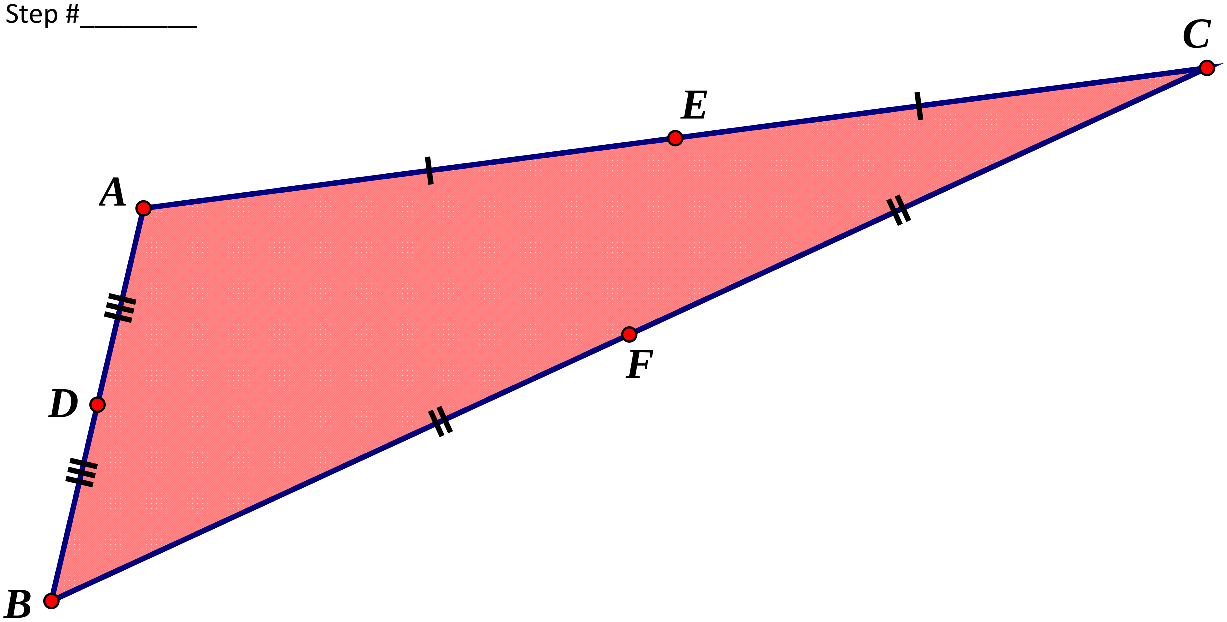
# Triangular Thinking 2 Card Match Cut-Outs

Cut out the following cards for use with the ***Card Match*** activity.  Each student will need one set of cut-outs and one activity sheet.

Step #________

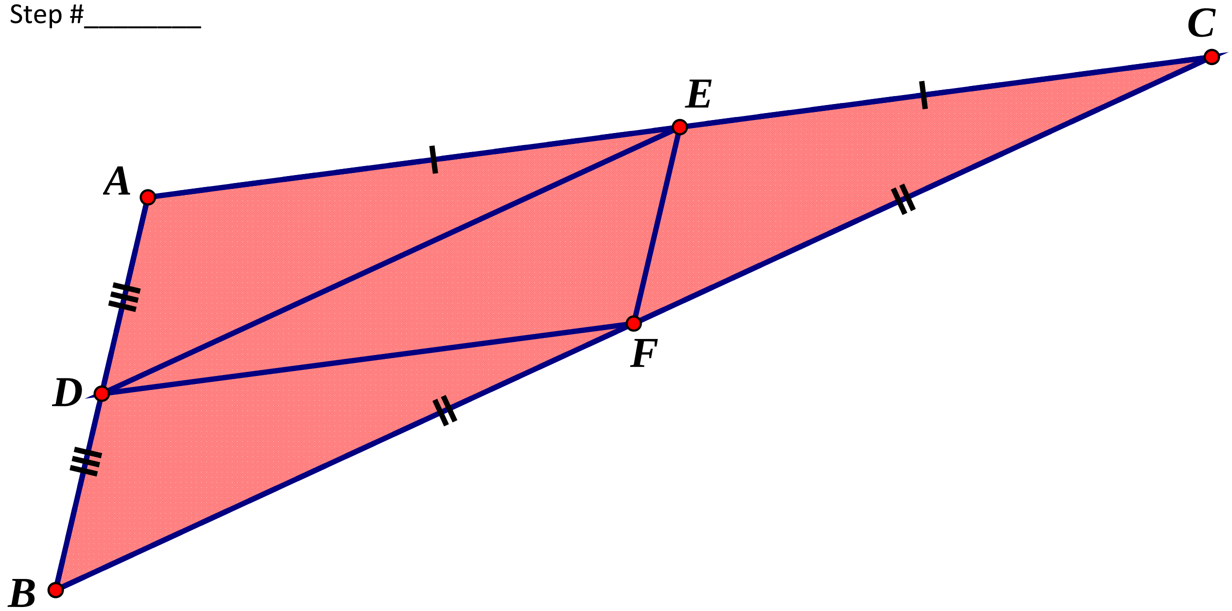



Step #________



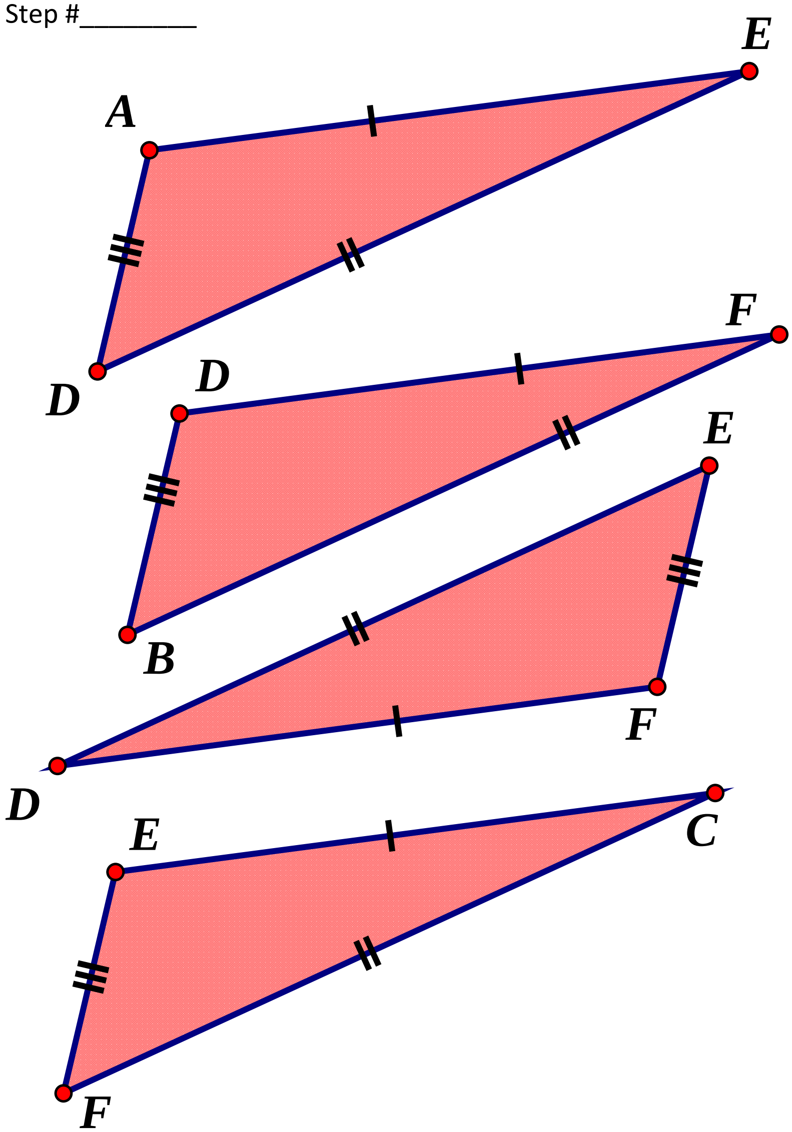
# Triangular Thinking 2 Card Match Cut-Outs

Step #________



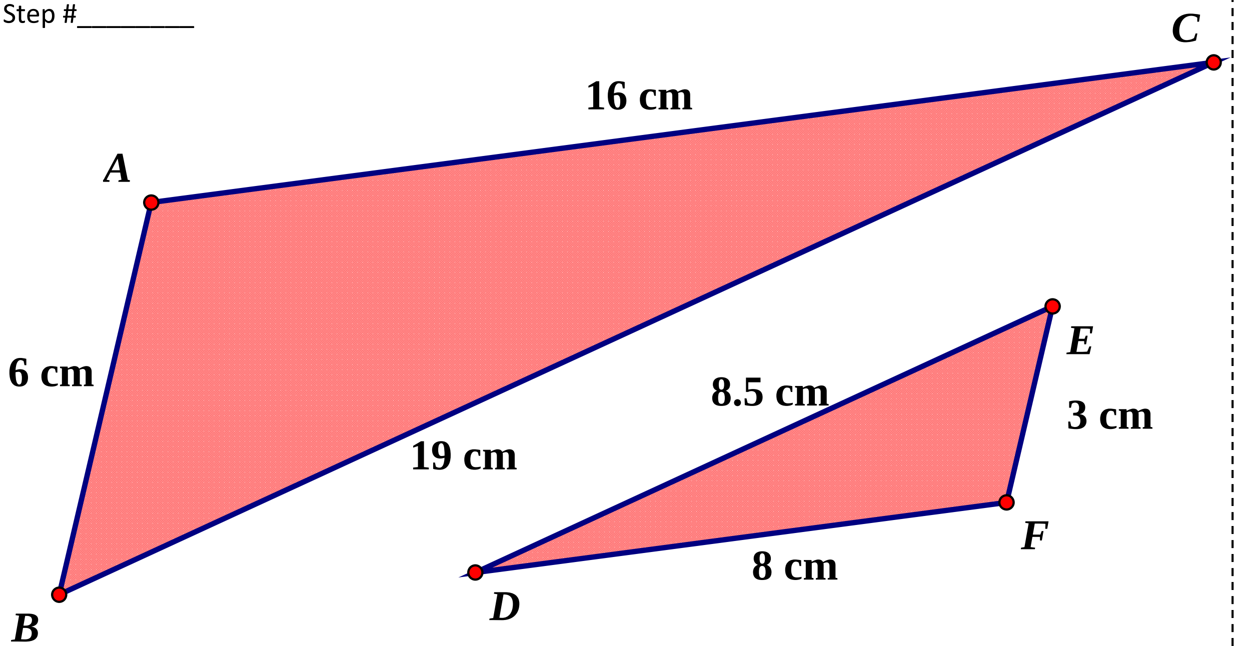
# Triangular Thinking 2 Card Match Cut-Outs

Step #________



# Triangular Thinking 2 Card Match Cut-Outs

Step #________



# Triangular Thinking 3 Composing Triangles

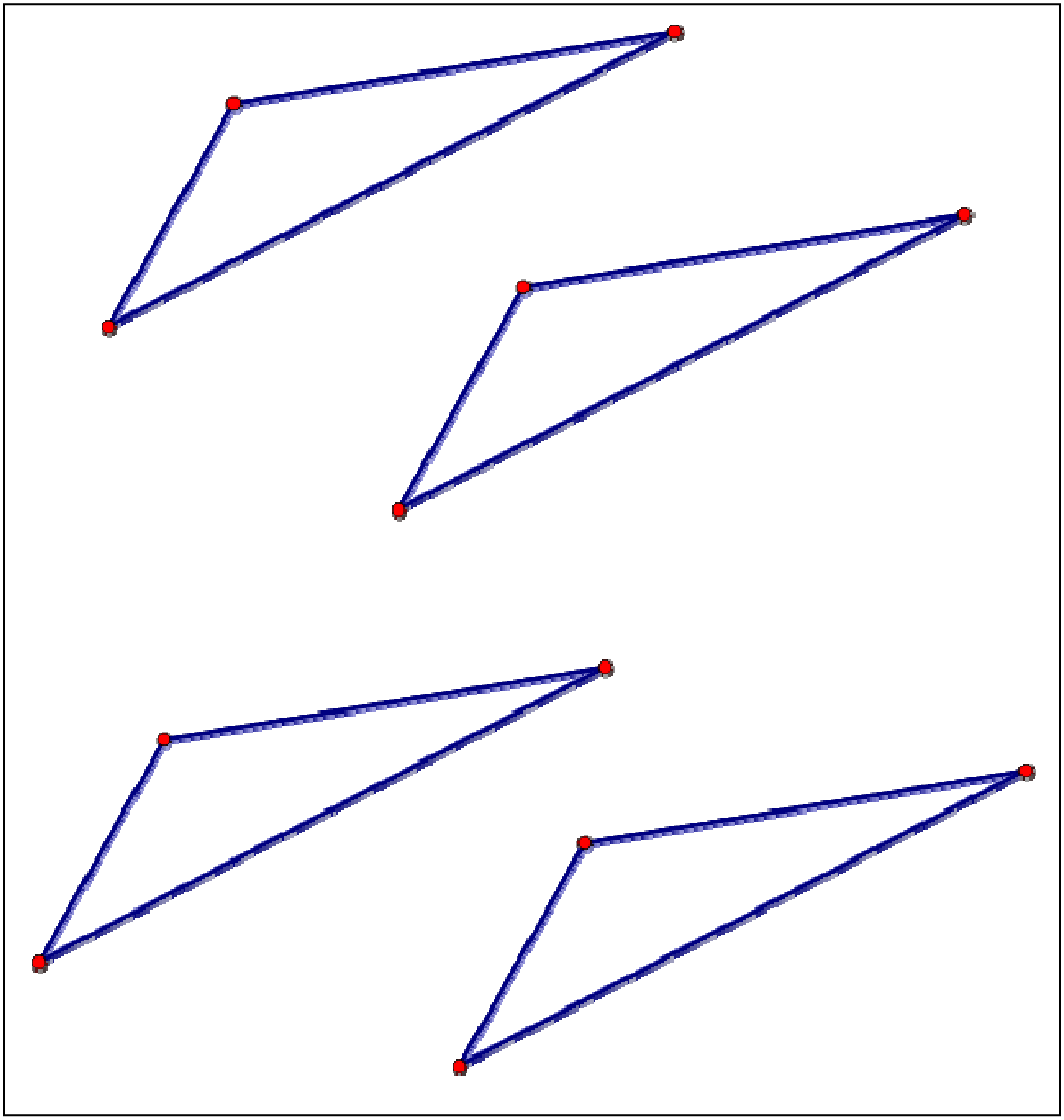
Each member of your group has been provided with a set of four congruent triangles. Complete the following.

1. Measure the side lengths of the triangles and mark their measurement on the triangles.
2. Arrange the four triangles into a single triangle.
3. What is the side length of the new triangle?
4. What is the relationship between the new triangle and the original four?
5. Compare your results with those of your group mates.
6. Complete the following sentence stem about the relationship between the triangles: ***If four congruent triangles are composed into a single triangle, then the new triangle is …***

# Triangular Thinking 3 Composing Triangles Cut-Outs

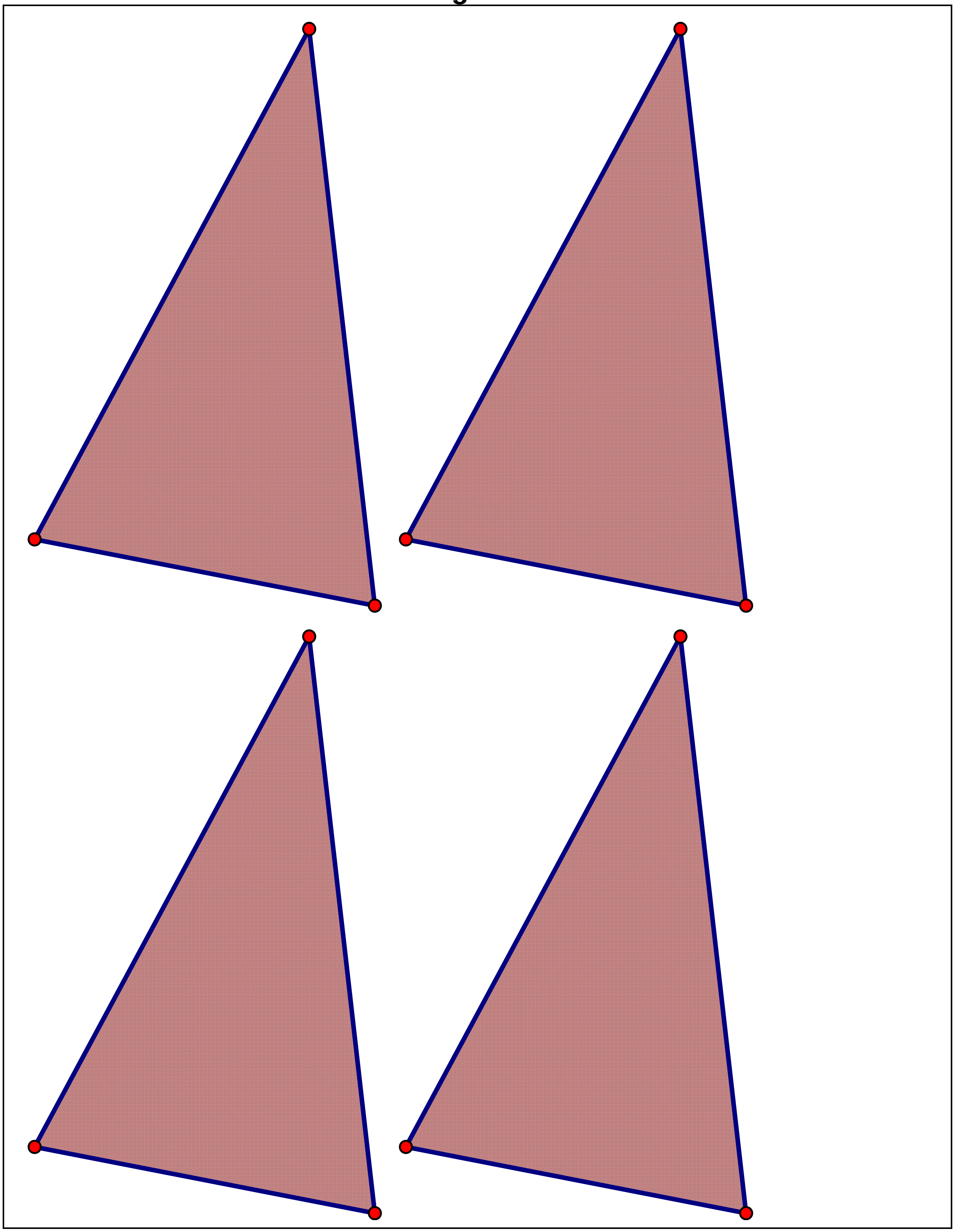
Five different sets of four congruent triangles are provided. This will allow groups of 5 to have five triangles to compare and draw conclusions.

**Triangle Set 1**



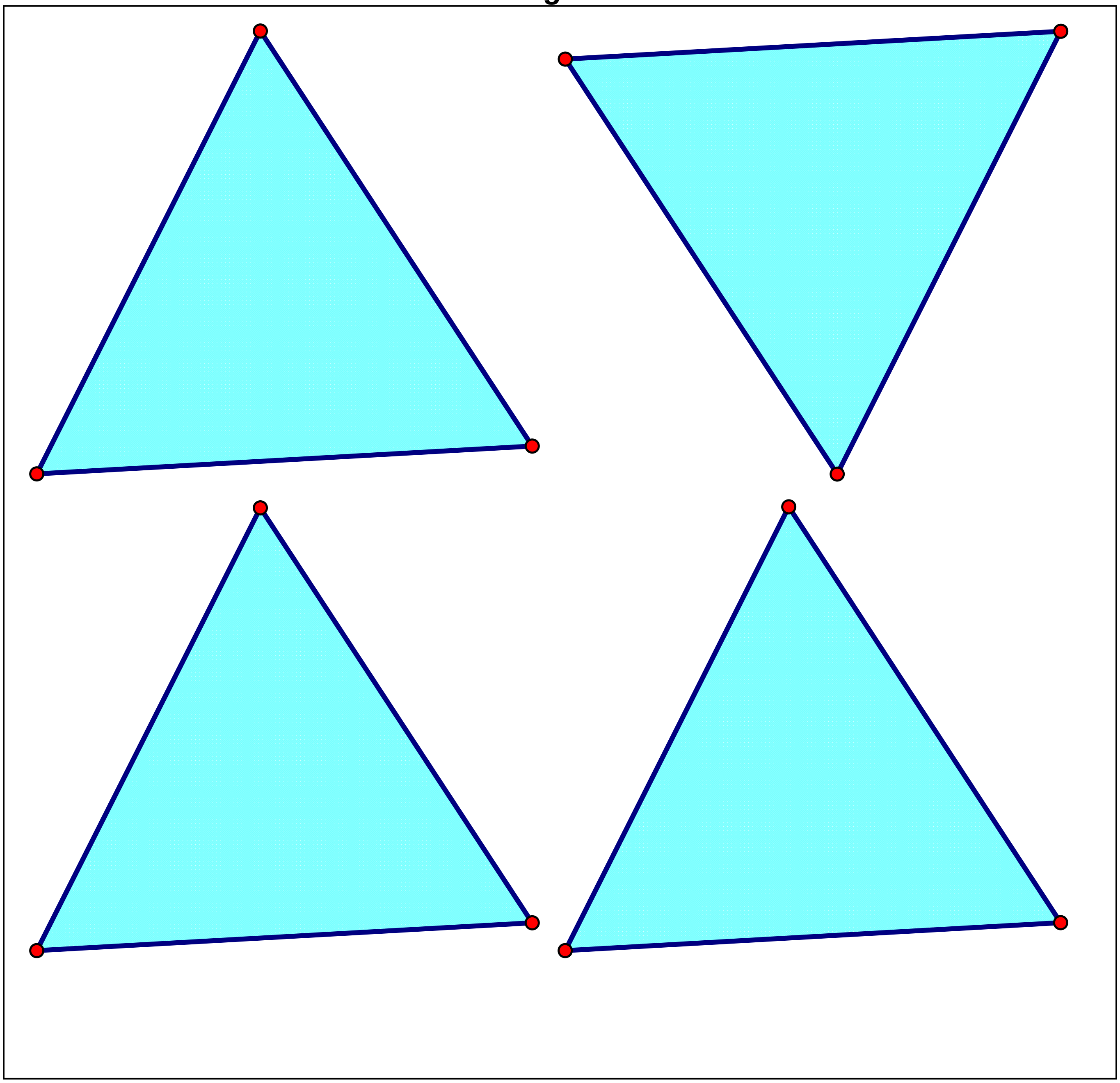
# Triangular Thinking 3 Composing Triangles Cut-Outs

## Triangle Set 2

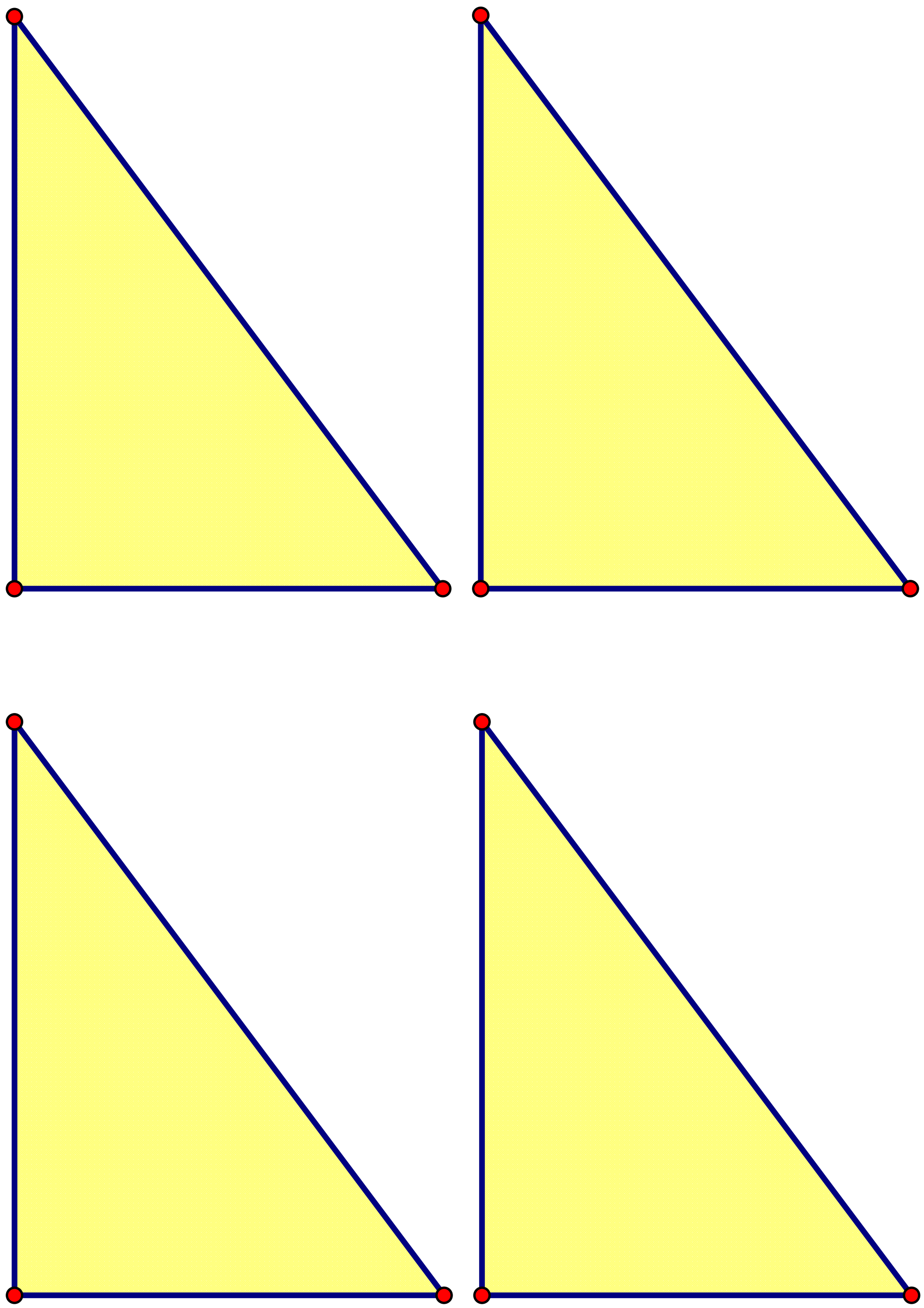


## Triangular Thinking 3 Composing Triangles Cut-Outs

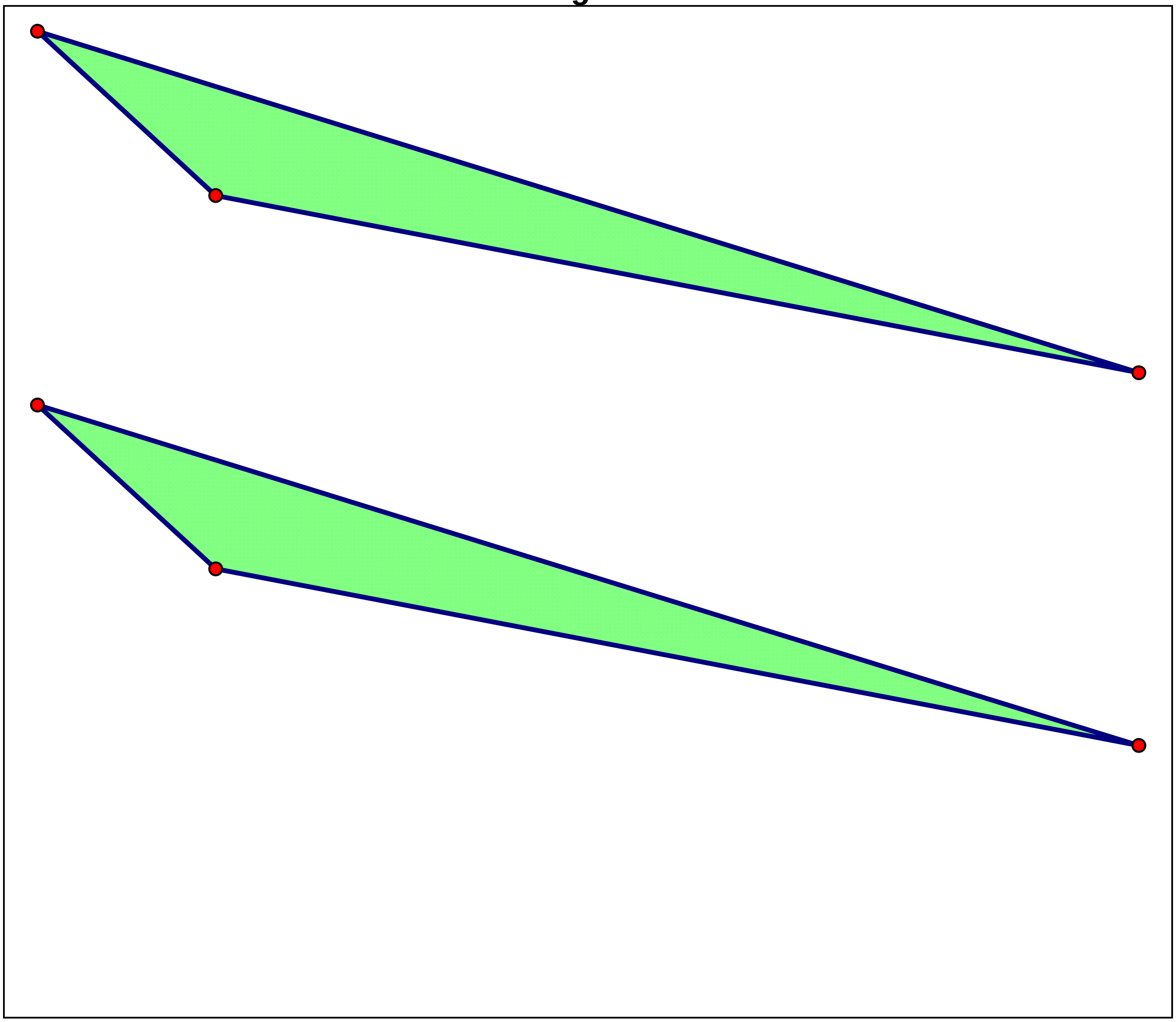
### Triangle Set 3



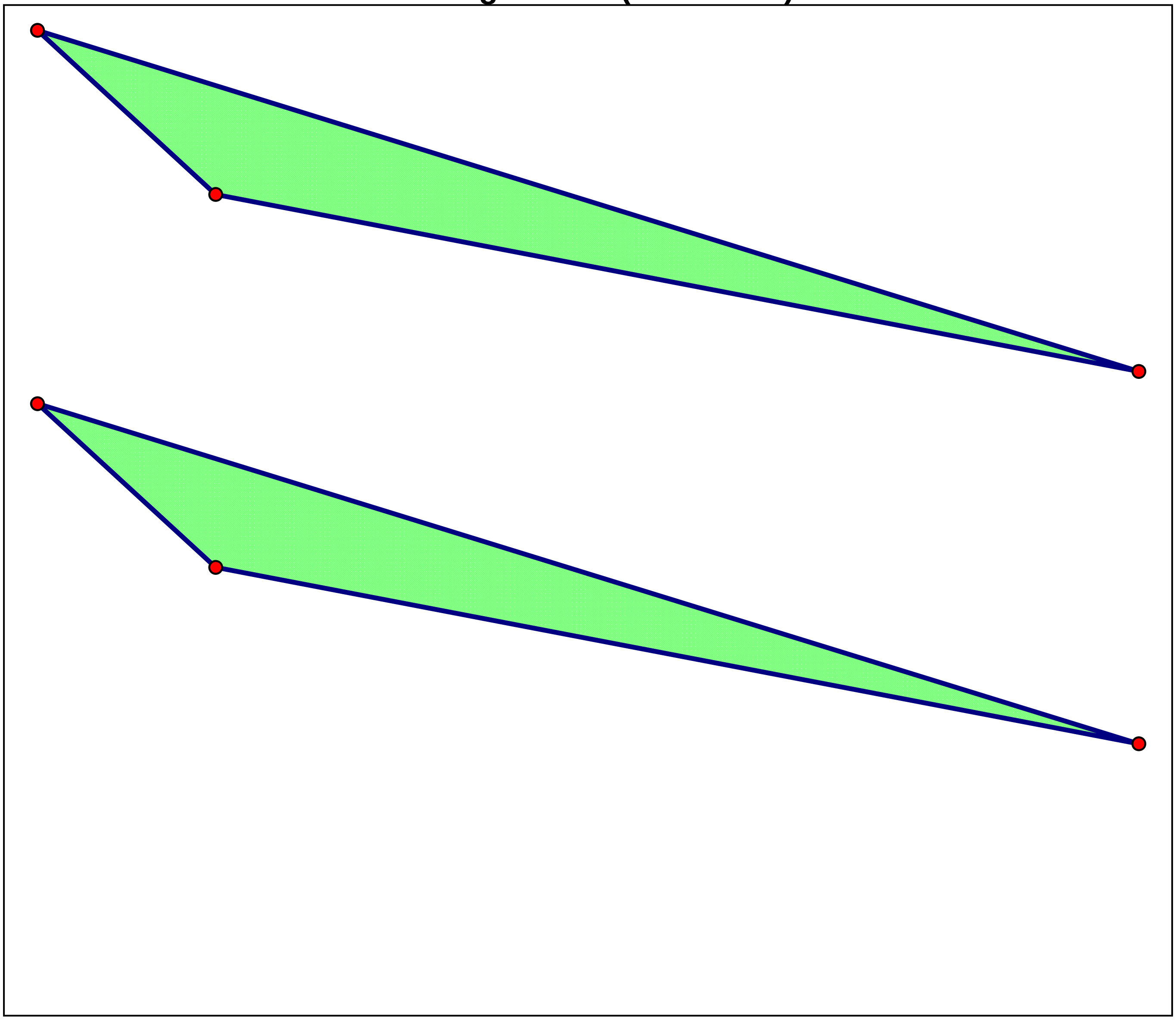
## Triangle Set 4



## Triangle Set 5



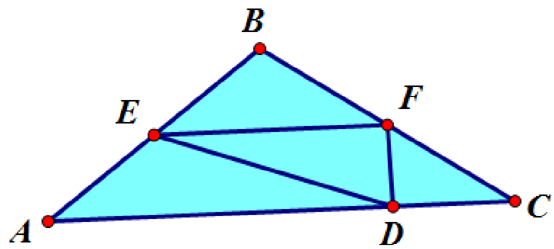
## Triangle Set 5 (continued)



# Triangular Thinking 4

Consider the following figure. Your classmate has come up with the following conjecture about this figure:

***Given:***




- ***Segment EF is parallel to segment AC***
- ***Segment ED is parallel to segment BC***
- ***Segment BF is congruent to segment FC***

***Conjecture:***

***Triangles AED, EBF, DFC, and FED are congruent.***

Prove (using an algebraic, flow chart, paragraph, or 2-column) that your friend's conjecture is correct.

# Triangular Thinking 5

In the ***Triangular Thinking*** activity, you showed that if you connected the midpoints of the sides of a triangle, the new triangle would be similar to the original.  Does a similar relationship hold for quadrilaterals?  If so, explain why this is true?  If not, create an example that demonstrates that the relationship does not hold.

# Circles from Different Angles

# Geometry EOC Success
## Lesson Plan Summary: *Circles from Different Angles*

**Topic:** Using the constructions of inscribed and central angles of circles to form conjectures

**CCRS:** In this lesson, the student will:
- make and validate geometric conjectures.
- develop and evaluate convincing arguments.
- use various types of reasoning.
- use mathematics as a language for reasoning, problem solving, making connections, and generalizing.

| | |
|---|---|
| **Content Objective:** The student uses geometric constructions to explore vocabulary and make, test, and justify conjectures relating to circles. | **Language Objective:**<br>C2(D) The student is expected to monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed.<br>C5(B) The student is expected to write using newly acquired basic vocabulary and content-based grade-level vocabulary. |
| **Vocabulary:** circle, chord, inscribed angle, central angle, intercepted arc, minor arc, major arc | **Prior Knowledge:** Students are expected to be able to measure angles and observe patterns. |

| RtI Tier I<br>Differentiation Activity | Instructional Phase | Enrichment Differentiation Activity |
|---|---|---|
| ***Mini-teach**: Classifying and measuring angles is introduced in 6th grade; however, it may have been a year or more since most students have had worked with this skill. A pre assessment of these skills should help determine which students would benefit from explicit instruction* in this area.<br><br>**Engage:**<br>• Students having difficulty with vocabulary will respond in their journal | **Engage:**<br>Students will be asked to read statements relating to measurements of central angles and intercepted arcs and apply their understanding of angles by hands of a clock on the ***Engage: Clocks & Angles*** handout.<br>Additional Materials: protractors<br><br>**Explore:**<br>Students will be provided inscribed and central angles in ***Circles from Different Angles 1: What's the Relationship?*** and make conjectures on the relationship between their measures.<br>Additional Materials: protractors, straight edge | • Students can extend their thinking by testing their conjecture on the sum of inscribed angles in inscribed stars in ***Circles From Different Angles 6***.<br><br>As another extension students are asked to make and justify a conjecture about similar triangles in ***Circles From Different Angles 7***. |

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<td rowspan="3">by completing three sentence stems on the Circles from Different Angles 3: Vocabulary Review. Provide the students with a circle that has been cut out and complete Circles from Different Angles 4: Vocabulary Illustrations.<br><br>Explore/Explain:<br>Groups may be assigned based on student level to allow more directed guidance where needed using a selection of the activities provided below.<br><br>• Give the students the drawings on Circles from Different Angles 5: Patty Paper™ and Angles. Have students trace the central angle, fold the paper in such a way as to bisect the angle and compare the measure to the inscribed angle.<br>Additional materials: Patty Paper™, protractor<br><br>• Think-aloud strategies can be used to work with struggling students. For example, to explain the thinking and reasoning needed to solve the problem, the teacher could say, “When I look at the drawing, I see a circle and two angles. The vertex of one angle is on the circle and the vertex of the other angle is at the center. The angle measures could not be equal.” Have the students explain what is observed.<br><br>*Explicit Instruction includes teaching components such as:<br>• clear modeling of the solution</td>
<td>In Circles From Different Angles 2: Exploring Intercepted Arcs and Measures of Angles students draw their own circles, inscribed angles and central angles and make conjectures.</td>
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<td>Explain:<br>Students will assume they are writing a geometry book. Their assignment is to write a conditional statement as a theorem, with a drawing at the end of What’s the Relationship?.<br><br>In Exploring Intercepted Arcs and Measures of Angles students will be asked to explain the relationships between the measures between central angles, inscribed angles and intercepted arcs.</td>
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<td>Formative Assessment:<br>Each student will receive a copy of <u>Closing the Loop: Do I Know the Relationship?</u> or the assessment could be projected for the entire class. The first question asked the students to construct inscribed angles from a given central angle measure. The second question asks students to justify the measure of an inscribed angle in a semi-circle.<br><br>Note:<br>Formative assessment items test concepts taught in the lesson and provide teachers valid information on whether students learned the concepts, principles and skills related to the lesson.<br><br>A transfer assessment question provides information on whether the students can take the concepts from the lesson and apply them in a novel situation.<br><br>Some situations may not lend themselves to transfer situations.</td>
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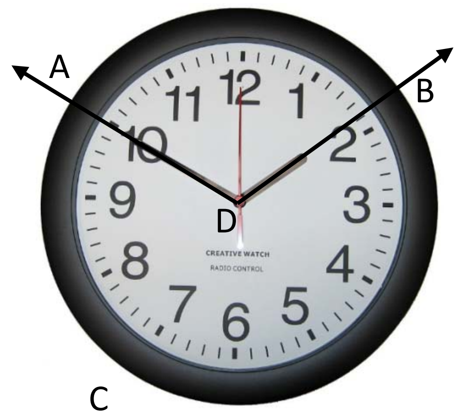
|---|---|---|
| specific to the problem;<br>• thinking the specific steps aloud during modeling;<br>• presenting multiple examples of the problem and applying the solution to the problems; and<br>• providing immediate corrective feedback to the students on their accuracy.<br><br>MSTAR Presenter's Guide pg 173 (2009) | | |

# Engage: Clocks and Angles

Name: ____________________________

Important vocabulary relating to a circle:

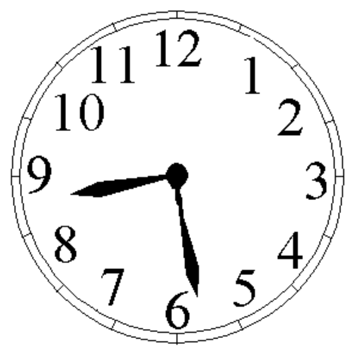
1. ∠*ADB* is a central angle because the vertex is at the center of the circle.
2. Arc *AB* is the intercepted arc of ∠*ADB*.
3. Arc *AB* is a minor arc because its measure is less than 180º.
4. Arc *ACB* is a major arc because the measure is greater than 180º or greater than the semicircle.
5. The measure of ∠*ADB* is about 114º.
6. The measure of arc *AB* is about 114º.




7. An angle is formed by the hands of the clock. Label the central angle and estimate the measure of the angle. ____________________

8. Label the major arc intercepted by the angle and estimate the measure of the arc.

________________________________



9. Label the minor arc formed by the hands of the clock and estimate the measure of the arc.

________________________________

10. Label the major arc intercepted by the angle and estimate the measure of the arc.

____________________

11. What is the measure of the central angle when the hands of the clock show 5:00?

12. Determine three possible times when the hands of the clock form 130º.

# Circles from Different Angles 1 – What's the Relationship?

Name ____________________________________

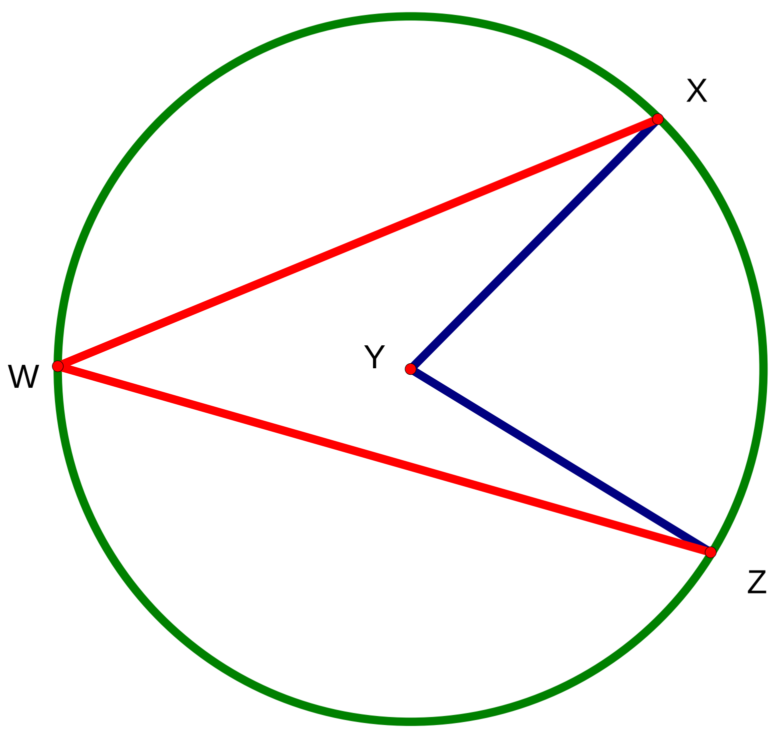
1. Given the circle with a central angle and an inscribed angle that intersects the same arc, as shown below.

- If $\angle XWZ$ is an inscribed angle, write a definition of an inscribed angle.

- Measure the angles with vertex *W* and vertex *Y*.

  $\angle W$ = _______________ $\angle Y$ = _________________

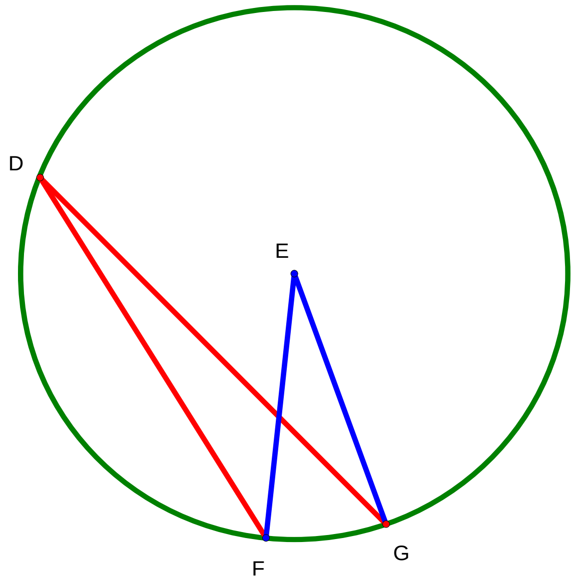
- One of the angles is a central angle and the other is an inscribed angle. What appears to be the relationship between the measures of the angles? In your own words, write a statement describing this relationship.



2. Is your description from question 1 valid for the figures below? Justify your answer.

1.

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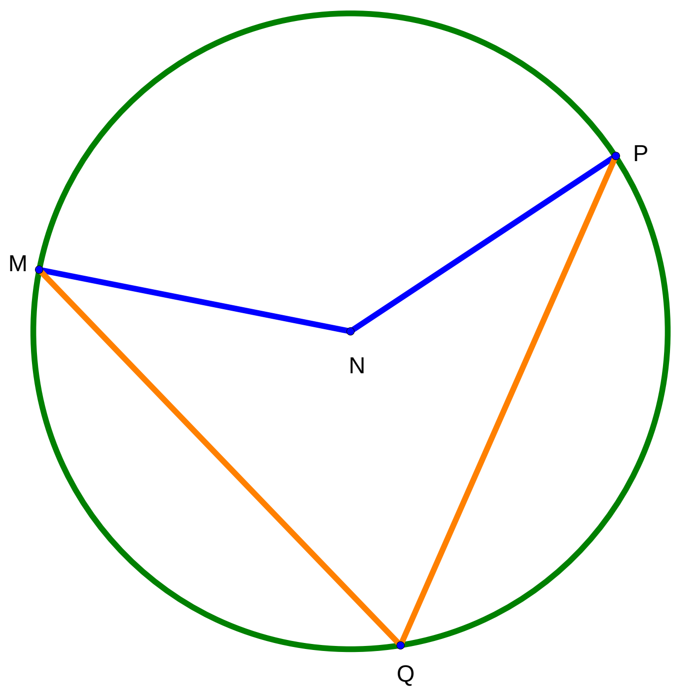


# Circles from Different Angles 1 – What's the Relationship?

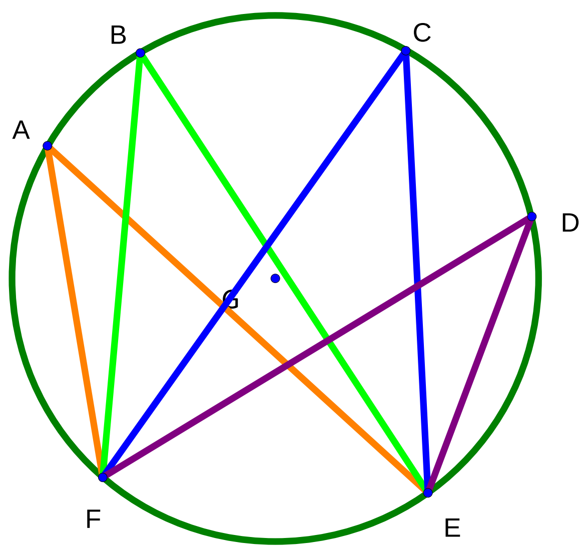
3. The measure of an arc is the same as the measure of its central angle. If the measure of the intercepted arc *FE* = 76º, using the circle with inscribed angles below, what are the measures of the following angles

$m\angle FAE$ = ________ $m\angle FBE$ = ________ $m\angle FCE$ = ________

$m\angle FDE$ = ________

Explain your thinking.

Draw the central angle that intersects the arc *FE*. What is the measure of this central angle? _____________




4. Suppose you are a writer of a geometry textbook. This new textbook is being written to help all students understand geometry. Write a theorem, as a conditional statement, stating the relationship between central and inscribed angles that intercept the same arc and draw a picture that illustrates this theorem.

## Closing the Loop: Do I Know the Relationship?

- Given a central angle of $130^\circ$. Construct the central angle and two inscribed angles that intercept the same arc. What must be true about the measure of the inscribed angles?

- A student in your class makes the following statement:

  *Any angle inscribed in a semicircle is a right angle.*

  Do you agree or disagree with this statement? What does the central angle of a semicircle look like? How do you measure that angle? What does an inscribed angle for a semicircle look like? Why is it special? Provide justification for your answer.

# Circles from Different Angles 2 – Exploring Intercepted Arcs and Measures of Angles

Name ___________________________________

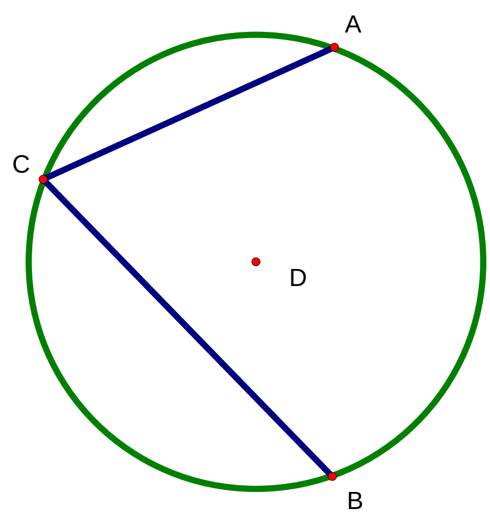
1. The measure of arc $AB = 140^{o}$.

   Measure inscribed $\angle ACB$ with a protractor. ____________

   Draw the central angle that has the same intercepted arc.

   What is the measure of this central $\angle ADB$? _________________________

   Is there a relationship between the measure of $\angle ACB$ and measure of $\angle ADB$?




2. Using a compass, draw a circle. Draw a central angle with a measure of $50^{o}$.

   Draw an inscribed angle that intercepts the same arc as the central angle.

   Measure the inscribed angle.

   What do you observe?

# Circles from Different Angles 2 – Exploring Intercepted Arcs and Measures of Angles

3. Draw another circle with a compass. Draw an inscribed angle of measure 60º.

   Draw a central angle that intercepts the same arc as the inscribed angle.

   Measure the central angle. What do you observe?

4. How would you explain to another student the relationship between the measures of a central angle and an inscribed angle that intercept the same arc?

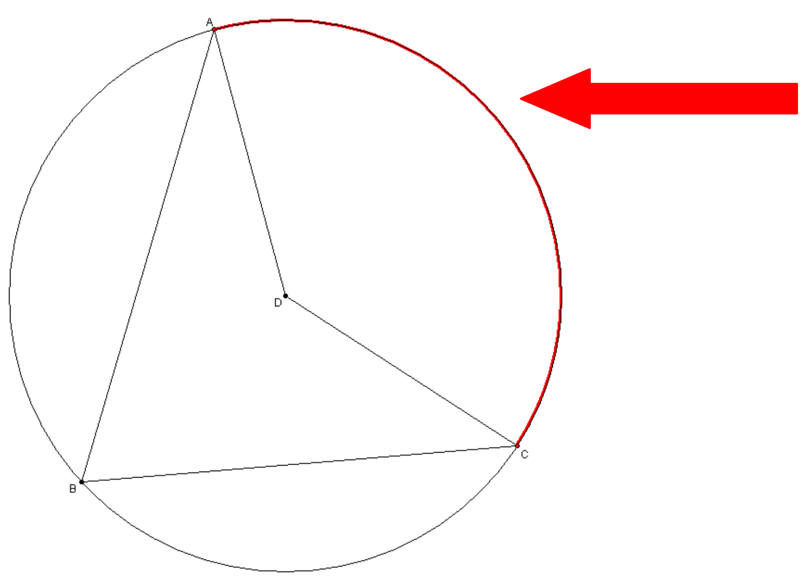
## In your journal complete the following sentence stems:

1. I can find the diameter of a circle by …
2. I can find the radius of a circle by …
3. I can find the center of a circle by …
4. Knowing the center of a circle, I can draw a central angle by …

# Circles from Different Angles 4: Vocabulary Illustration

Use the word bank to label the parts of the circle illustrated in RED

| | | | |
|---|---|---|---|
| **Acute angle** | **Major arc** | **Intercepted arc** | **Minor arc** |
| **Semi-circle** | **Central angle** | **Arc** | **Vertex** |
| **Arc** | **Arc** | | |

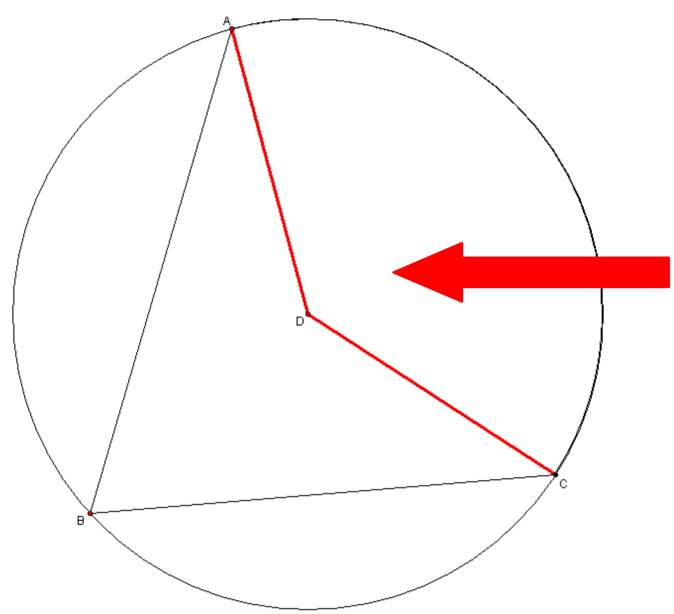



1. ______________________

2. ______________________

3. ______________________

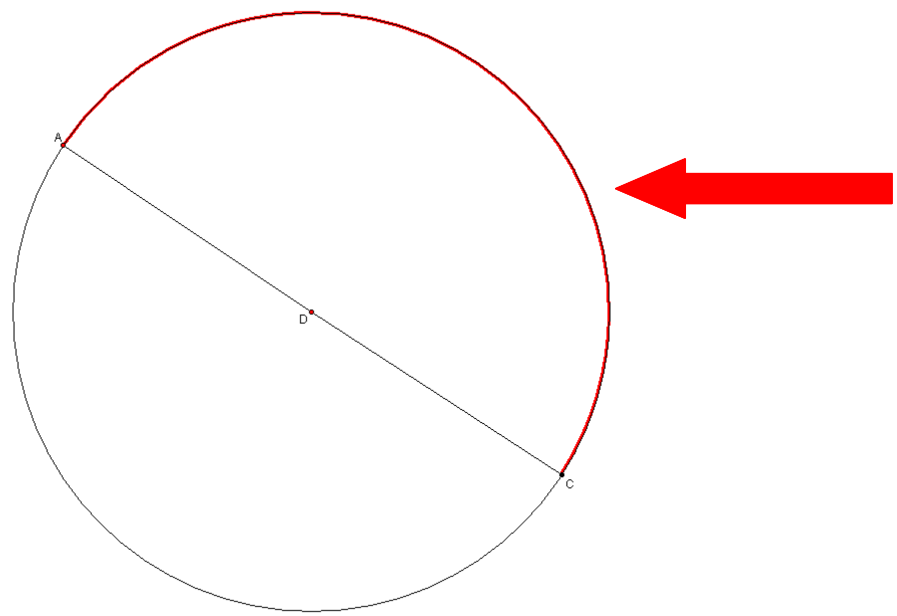
## Circles from Different Angles 4: Vocabulary Illustration




4. ______________________________

5. ______________________________

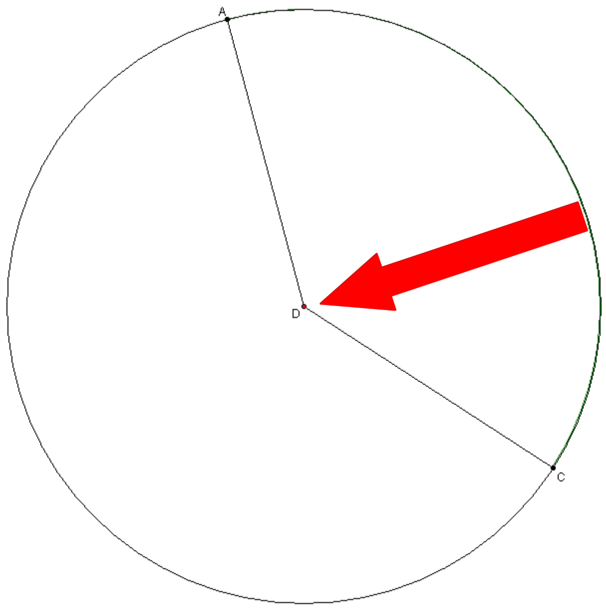
# Circles from Different Angles 4: Vocabulary Illustration




6. ______________________________

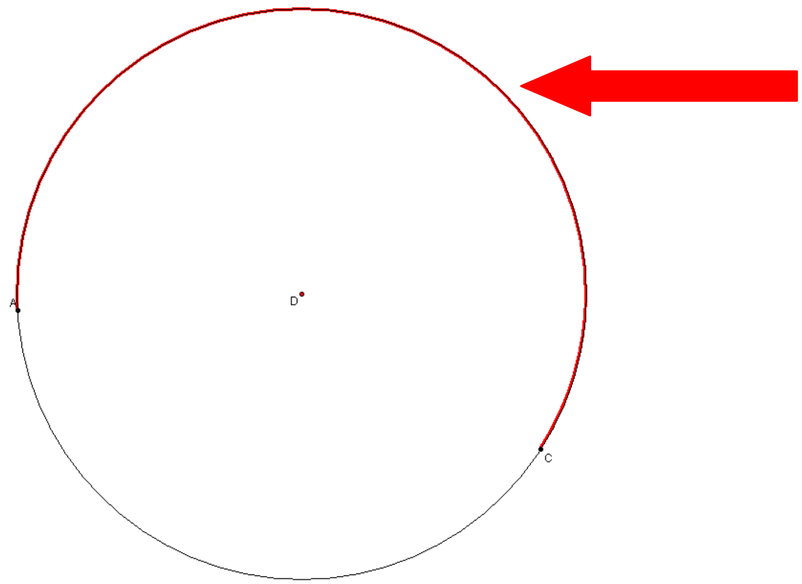
7. ______________________________

## Circles from Different Angles 4: Vocabulary Illustration




8. ____________________

# Circles from Different Angles 4: Vocabulary Illustration



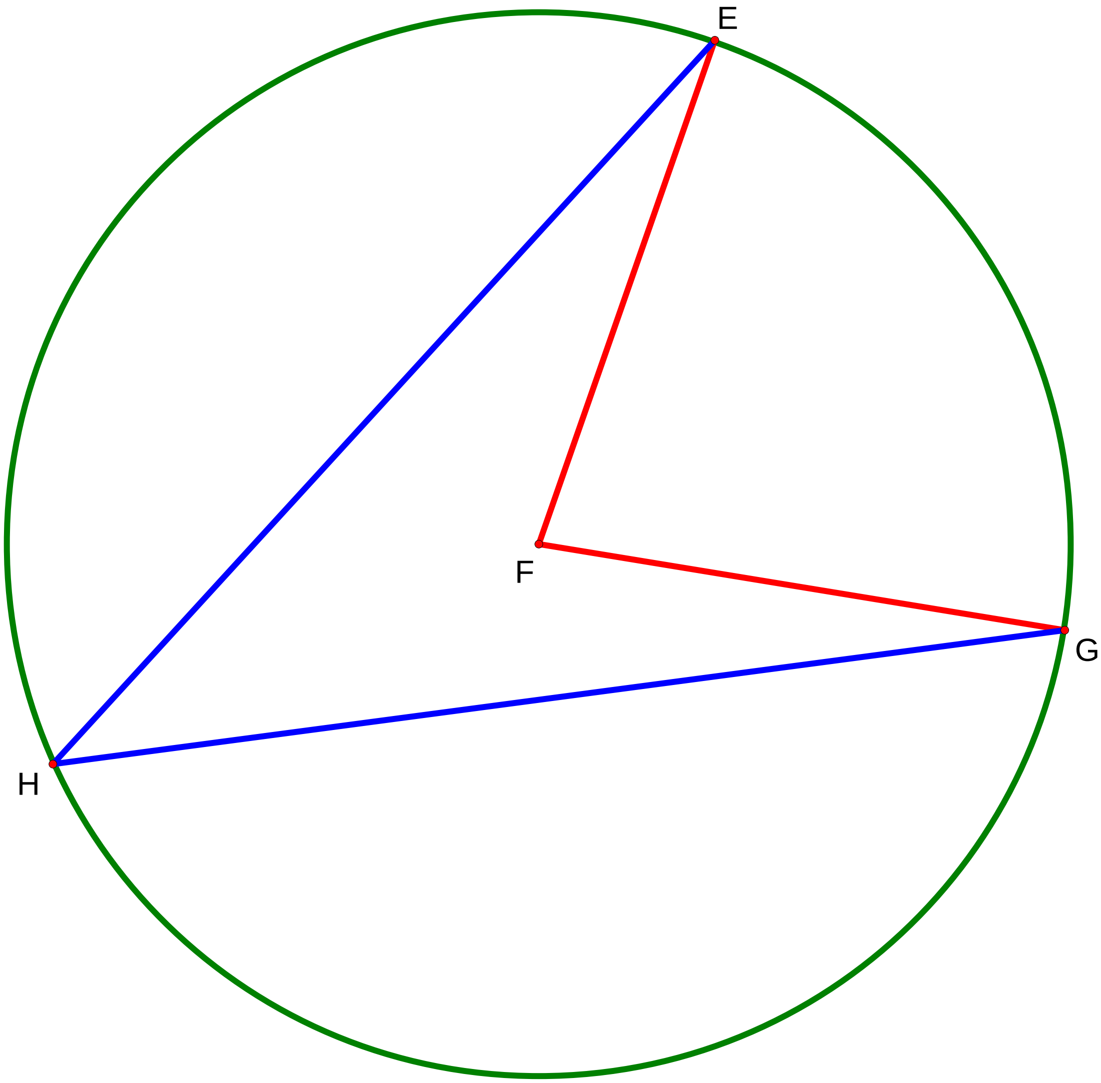

9. ______________________________

10. ______________________________

# Circles from Different Angles 5 – Patty Paper™ and Angles

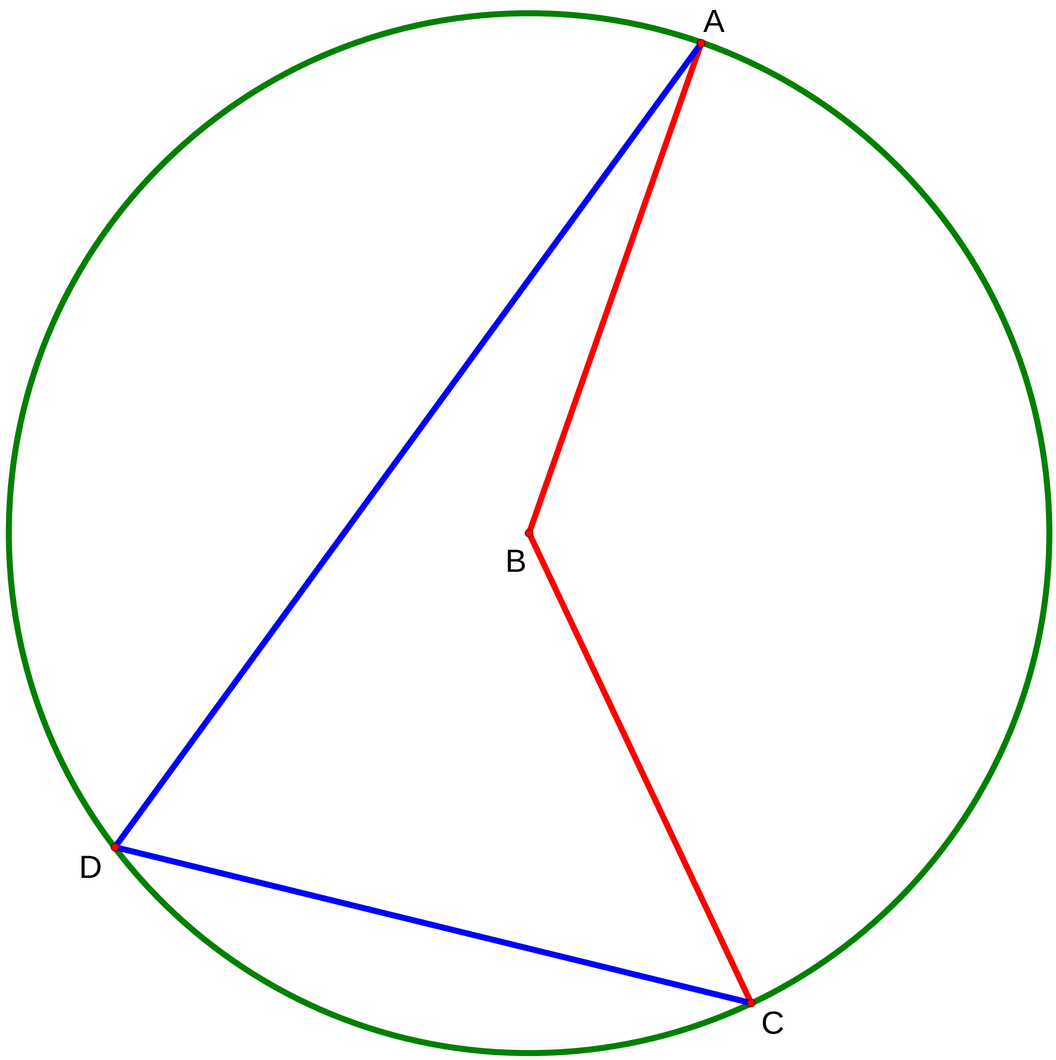
Name ___________________________________

1. Trace the central ∠*EFG* on Patty Paper™. Fold the Patty Paper™ in such a way that the segment *EF* is mapped onto segment *FG.* Crease the Patty Paper™. Use a protractor to measure and compare the bisected angle to the inscribed ∠*EHG.*



## Circles from Different Angles 5 – Patty Paper™ and Angles

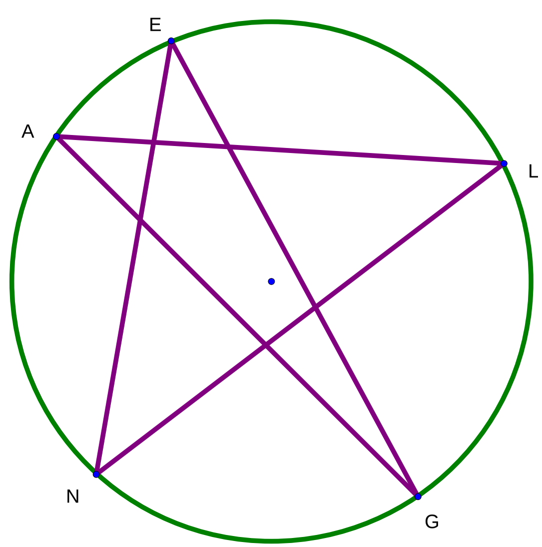
2. Repeating the process described in problem 1 use the Patty Paper™ to compare the measures of the central and inscribed angles below.

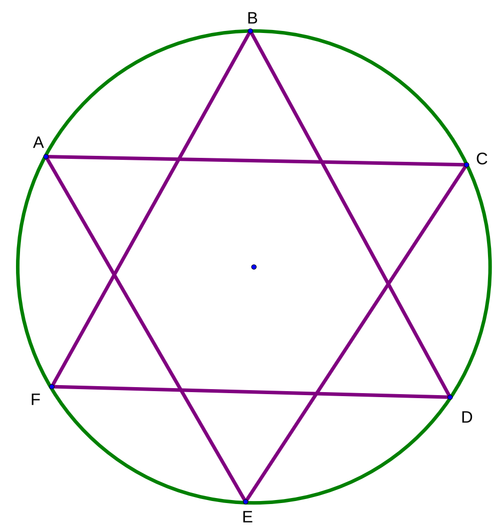



3. Write a conjecture on what has been observed.

## Circles from Different Angles 6

**The two inscribed stars are made using inscribed angles. The sum of the inscribed angles on one of the stars is 180º. The sum of the inscribed angles on the other is 360º. Using the given information, justify the sum of the inscribed angles for each. Write a conjecture based on your observations. Test your conjecture with other inscribed stars.**

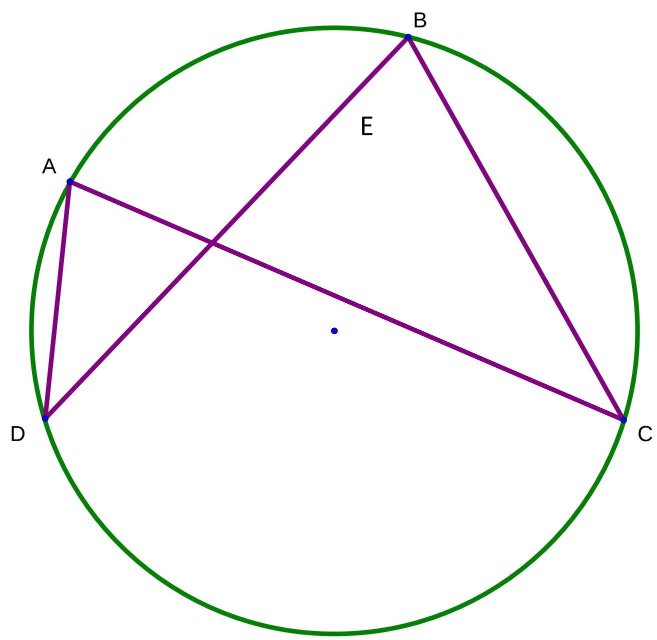


**Points *A, B, C and D* are randomly placed on the circle. Chords are drawn as shown, forming two triangles. Notice segment AC and segment *BD* intercept at point *E.***

1. ***Write a paragraph explaining why the triangles are similar.***
2. ***When would the triangles be congruent?***

**Justify your answer. Write a conjecture that applies to this problem situation.**

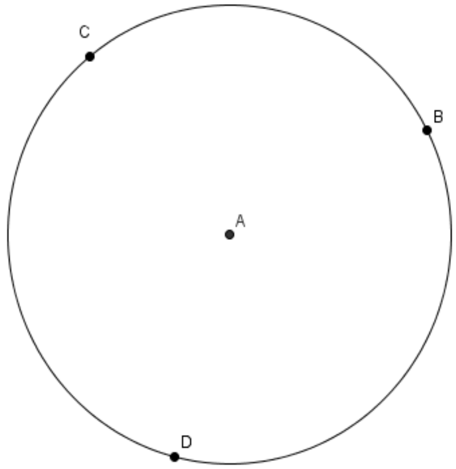


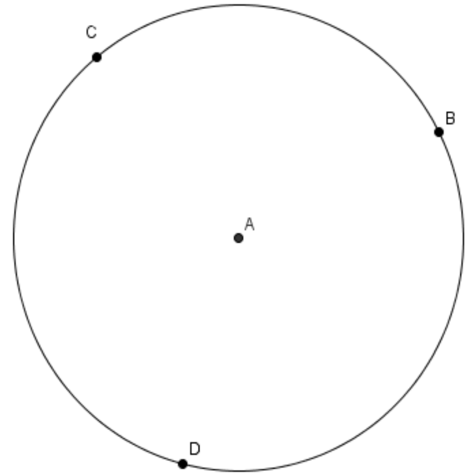
## Circles and Angles Journal Activity

1. Cut circle cards on dotted line and tape/glue onto journal page.
2. Draw a visual representation of the given statements on the circles.
3. After having drawn the representation, write the statement under the tab created with the circle cards.
4. Each circle will have at least one statement, but more than one statement may apply. All statements will be used.

Circle Statements

| | |
|---|---|
| $\angle CAB$ is a central angle. | $\overset{\frown}{CB}$ is a minor arc. |
| $\angle CDB$ is an inscribed angle. | $\overset{\frown}{CB}$ is the intercepted arc of $\angle CAB$ |
| $\angle CAB$ is an acute angle. | $\overset{\frown}{CB}$ is the intercepted arc of $\angle CDB$ |
| $\angle BAD$ is a central angle. | $\overset{\frown}{CDB}$ is a major arc. |
| $\angle CDB$ is an acute angle. | $\angle BCD$ is an inscribed angle. |


C
B
A
D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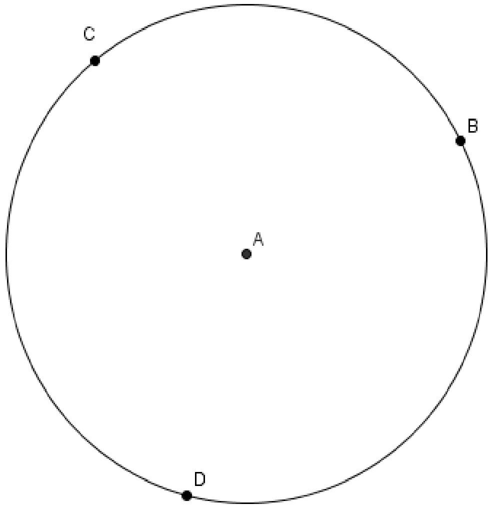
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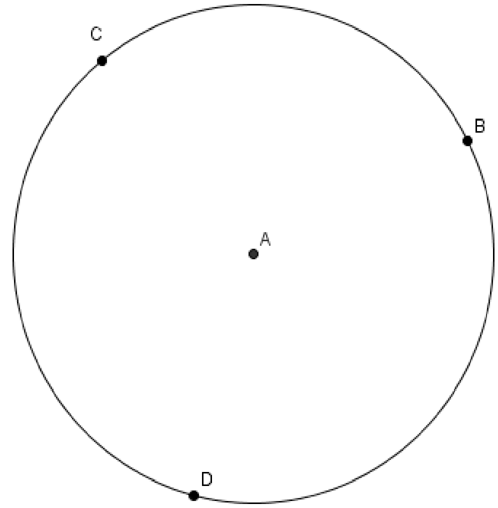
C
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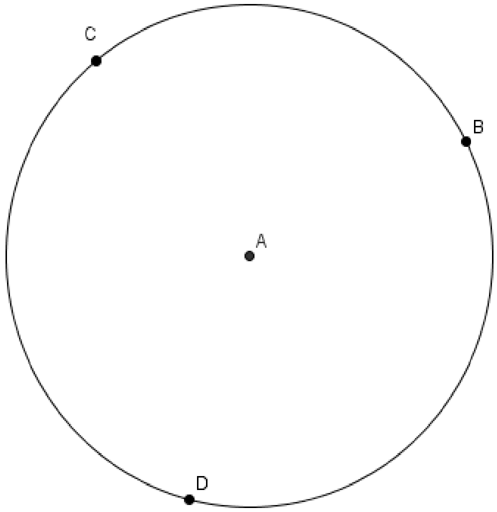
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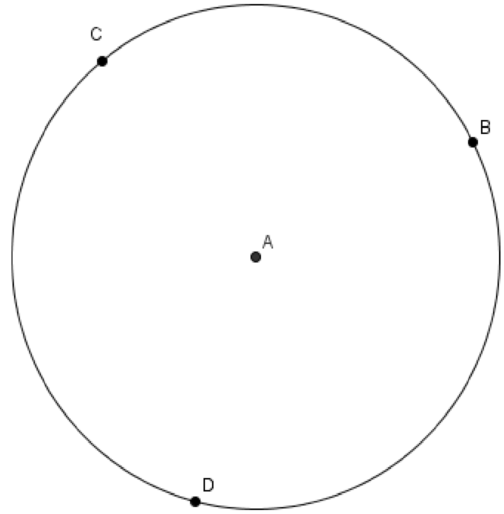

C
B
A
D

# Cylindrical Studies

# Geometry EOC Success
## Lesson Plan Summary: *Cylindrical Studies*

**Topic:** Describing the effect on volume when the dimensions of a cylinder are changed

**CCRS:** In this lesson, the student will:

- make, test, and use conjectures about one-, two-, and three-dimensional figures and their properties.
- determine the surface area and volume of three-dimensional figures.
- use mathematics as a language for reasoning, problem solving, making connections, and generalizing.

| | |
|---|---|
| **Content Objective:** The student uses cylinders to develop an understanding of the effects of dimensional change in a three-dimensional figure. | **Language Objective:**<br>C1(C) The student is expected to use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary<br>C3(D) The student is expected to speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency |
| **Vocabulary:** volume, cylinder, rectangular prism | **Prior Knowledge:** Students are expected to be familiar with the 3-dimensional volume formula ***V=Bh***. |

| RtI Tier I Differentiation Activity | Instructional Phase | Enrichment Differentiation Activity |
|---|---|---|
| **Engage**<br><br>• **Mini-teach**: The teacher will use a pre-assessment to determine students who will benefit from explicit instruction* in computing the volume of cylinders using the formula V=Bh.<br><br>• Students may be provided with a drawing of a cylinder and asked to label the | **Engage: Journal assignment**<br>Students will be given a copy of ***Engage: The Best Buy*** in which they will be asked to explain how to decide between two cylindrical packages of candy.<br>Additional Materials: ***Engage: Cylindrical Studies - Cylindrical Cut-Outs***, Student Journals<br><br>**Explore/Explain:**<br>***Cylindrical Studies 1*** **–** Students will be asked to think about three cylinders (one in which the radius is greater, another with greater height, and the last with equal radius | • Students may be asked to show algebraically that increasing the radius will cause a greater increase in the volume than increasing the height by the same factor.<br><br>• ***Cylindrical Studies 2*** **–** Students will be asked to come up with cylinders that have a volume 9 times greater than a given cylinder by changing |

<table>
<tr>
<td rowspan="2">radius and height.<br><br>• Students could be asked to show the base of the cylinder and asked to explain how this information connects to the volume formula for the 3-dimensional figures.<br><br>• Students who do not have an understanding of the volume of a cylinder might be asked to examine the MSTAR GATAR activity Volume of Cylinder.<br><br>• Explore/Explain – Students who have difficulty working with dimensional change in 3-dimensions may be asked to examine 2-dimensional figures instead. Students should be asked to examine proportional change and non-proportional change.<br><br>*Explicit Instruction includes teaching components such as:<br>• clear modeling of the solution specific to the problem;<br>• thinking the specific steps aloud during modeling;<br>• presenting multiple examples of the problem and applying the solution to the problems; and<br>• providing immediate corrective feedback to the students on their accuracy.<br>MSTAR Presenter's Guide pg 173 (2009)</td>
<td>and height). After experimenting with scale factor changes of the radius and height of these three cylinders, students will predict which dimensional increase will cause a greater increase in volume. Students will then explain to an absent classmate how the volume formula supports their conjecture.</td>
<td rowspan="2">only one dimension at a time.</td>
</tr>
<tr>
<td>Formative Assessment:<br>Provide students with a copy of Closing the Loop: Cylindrical Studies I. This problem involves a situation based on cylinders in the unique form of a swimming pool.<br><br>Note:<br>Formative assessment items test concepts taught in the lesson and provide teachers valid information on whether students learned the concepts, principles and skills related to the lesson.<br>A transfer assessment question provides information on whether the students can take the concepts from the lesson and apply them in a novel situation.<br><br>Closing the Loop: Cylindrical Studies II models a transfer assessment for this lesson. Students will be asked to repeat the Cylindrical Studies activity with the cylinder replaced with a rectangular prism whose base is square. Many times state assessments require transfer of knowledge, therefore; both types of questions should be used. It is necessary to remember transfer items require students have a wide range of examples; these provide the background knowledge essential for transfer of information.</td>
</tr>
</table>

# Engage: The Best Buy

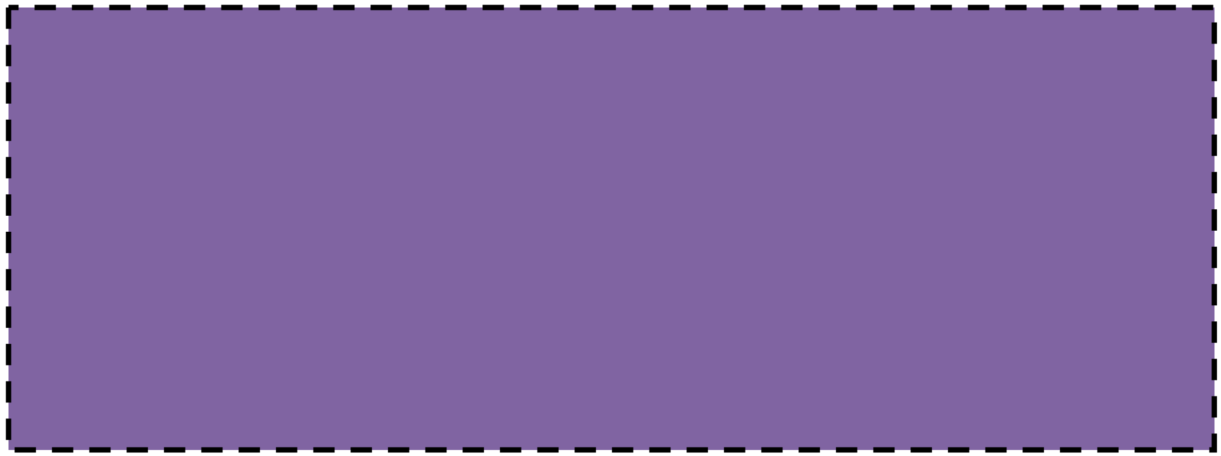
In preparation for the Halloween party that you are having this weekend, you stop by the store to pick up some jelly beans. The store has two containers of jelly beans from which to choose for the same price. One is a cylinder that is 8 inches tall and has a radius of 2 inches. The other is a cylinder 2 inches tall with a radius of 4 inches. José says the taller one is the better buy. Do you agree? In your journal, write your response to José.

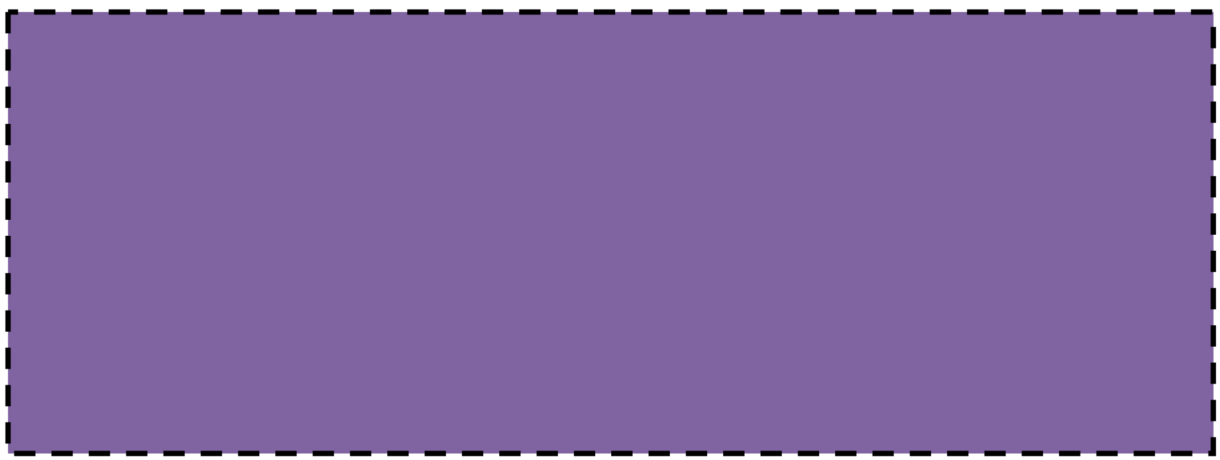
# Engage: Cylindrical Studies – Cylinder Cut-Outs

To create the cylinder that is 8 inches tall with radius 2 inches, print two copies and tape long sides together.

## Engage: Cylindrical Studies – Cylinder Cut-Outs

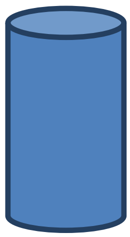
To create a cylinder that is 2 inches tall with a radius of 4 inches, print two copies of these rectangles (for a total of 4 rectangles) and tape short sides together.





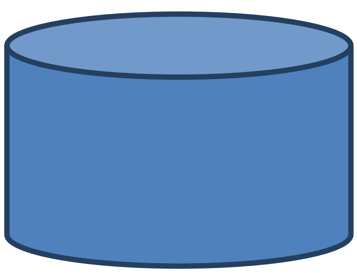
# Cylindrical Studies 1

1. Consider a cylinder with radius 1 cm and height 5 cm. If we are allowed to double only the radius or the height, which will cause the greater increase in the volume?



2. Consider a cylinder with radius 5 cm and height 1 cm. If we are allowed to change only one of the dimensions, which one will cause the greater increase in the volume?

3. Consider a cylinder with radius 5 cm and height 5 cm. If we are allowed to double only one of the dimensions, which one will cause the greater increase in the volume?



4. Based on the previous three problems, make a conjecture about which dimension to increase by a scale factor if you want the greater increase in the volume.

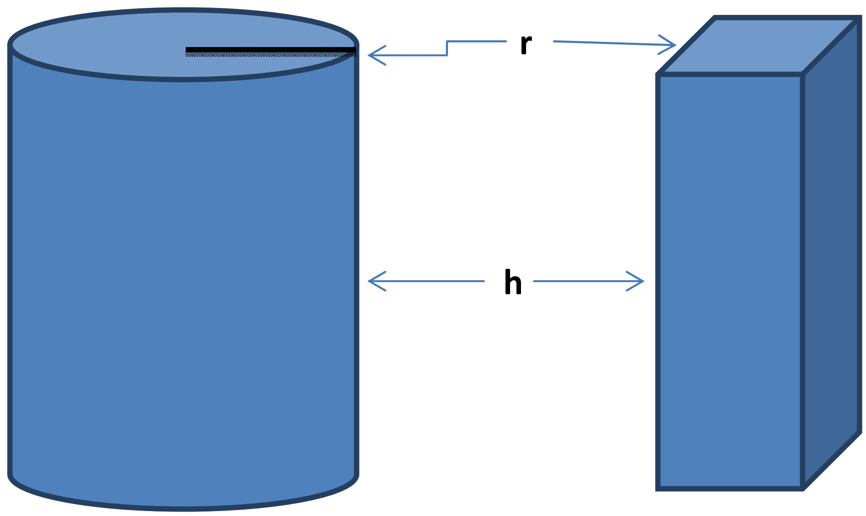
5. Josh missed geometry class today. Explain to Josh why the formula for determining the volume of the cylinder makes you believe your conjecture is true.

# Closing the Loop: Cylindrical Studies 1

1. John's brother is in elementary school, he has a circular swimming pool with a radius of 5 feet and a depth of 3 feet. John is in high school and his parents have agreed to purchase a circular above ground pool that will hold only 4 times the amount of water in his brother's pool. What are the dimensions of the pool he can by if the store only has pools that are deeper or pools that are wider? Which change will result in the most reasonable dimensions for a new pool?

## Closing the Loop: Cylindrical Studies 2

1. Compare the volume formulas for a cylinder with radius *r* and height *h* and a rectangular prism of height *h* and square base with sides of length *r*.




Figures are not drawn to scale.

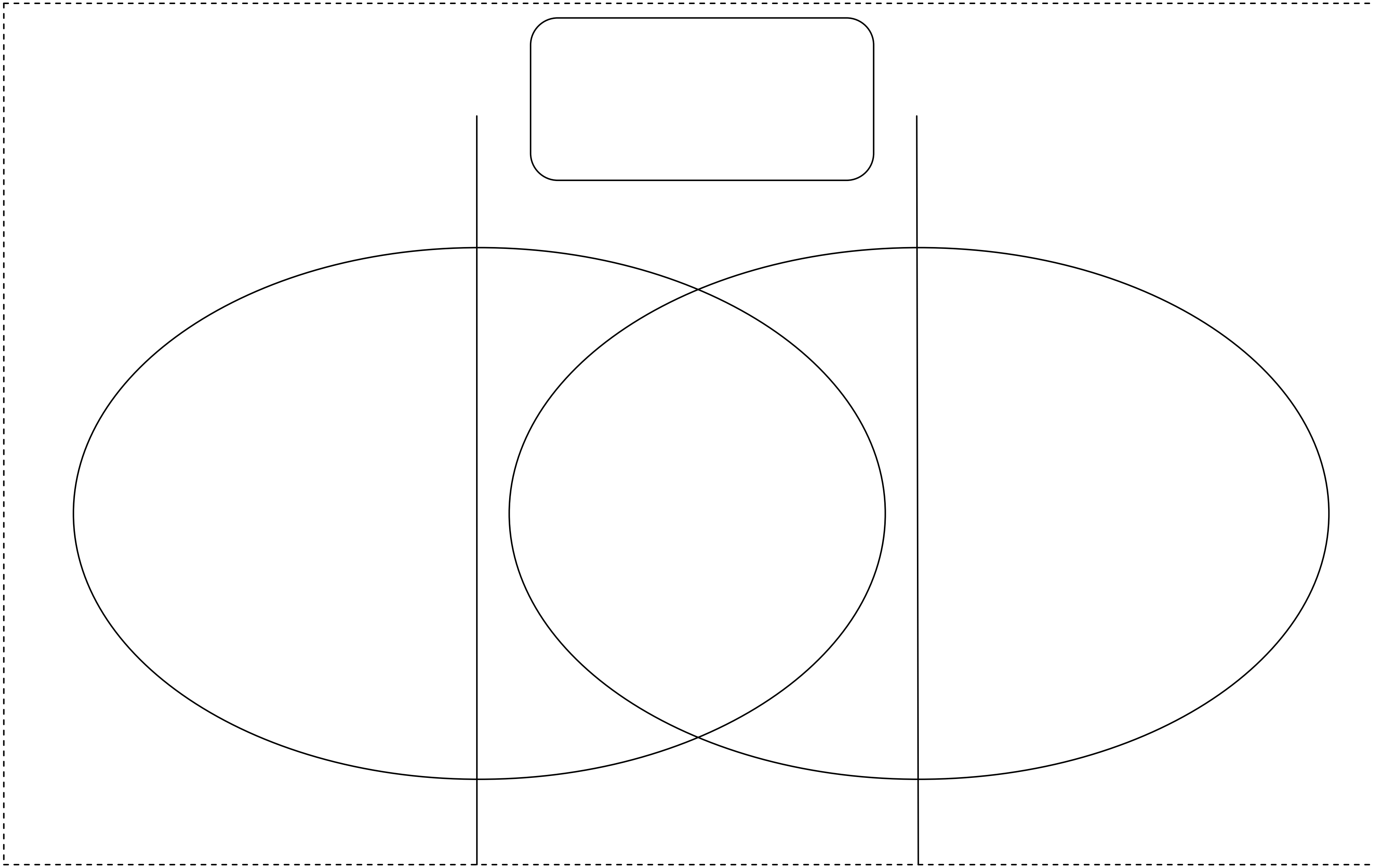
2. Based on your comparison in the above problem and on your conjecture from the ***Cylindrical Studies*** activity, if you wanted to increase the volume of a square prism (a rectangular prism whose base is a square) by the greater amount, would you increase the height or the side length of the square prism? Explain.

# Cylindrical Studies 2

1. As head designer at a container store, you are asked to create a line of cylindrical containers that all have volume 9 times as much as one that is a best seller at the store. The best selling cylinder is 10 cm tall and has a radius of 4 cm. If the company's production equipment requires that the dimensions of the cylinders be in whole centimeters and that each new container result from changing only one dimension of the best seller, describe 4 new cylindrical containers that you want to recommend for the new product line.

2. Suppose the container store is restricted to changing only one dimension – the height or the radius. If the store owner wants you to choose based on which dimension can be changed the least, which dimension would you recommend go unchanged? Explain.

# Cylindrical Studies - Journal Activity



# Segmented Circles

## Geometry EOC Success
## Lesson Plan Summary: *Segmented Circles*

**Topic:** Using circles and chords to form conjectures.

**CCRS:** In this lesson, the student will:

- make and validate geometric conjectures.
- develop and evaluate convincing arguments.
- use various types of reasoning.
- use mathematics as a language for reasoning, problem solving, making connections, and generalizing.

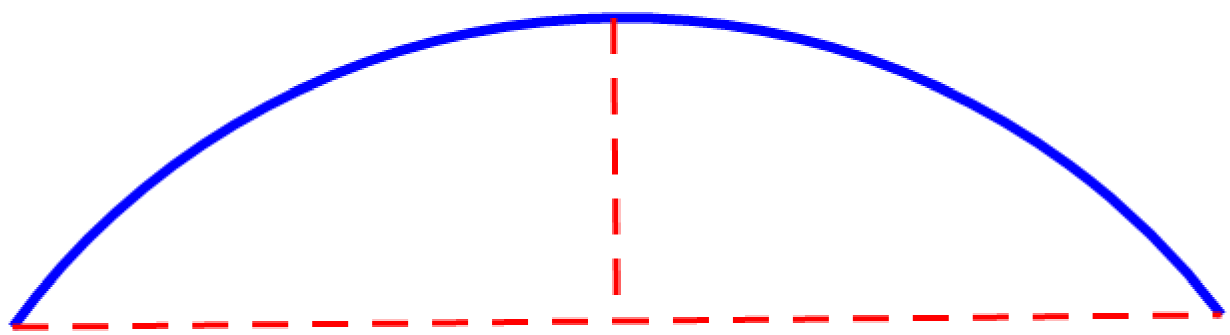
| | |
|---|---|
| **Content Objective:** The student uses geometric constructions to make justify conjectures. | **Language Objective:**<br>C2(C) The student is expected to learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.<br>C3(E) The student is expected to share information in cooperative learning interactions. |
| **Vocabulary:** chord, midpoint, segment, radius, perpendicular bisector | **Prior Knowledge:**<br>*For Enrichment* – Students are expected to know that the measure of an inscribed angle is determined by the measure of the arc that subtends that angle (***Inscribed Angle Theorem),*** and if corresponding angles in two triangles are congruent, then there is a proportional relationship between the triangles.<br>*For Formative Assessment* – Students are expected to know that the perpendicular bisector of a chord will pass through the center of the circle. |

| RtI Tier I Differentiation Activity | Instructional Phase | Enrichment Differentiation Activity |
|---|---|---|
| **Engage:**<br><br>• **Mini-teach**: The teacher will use a pre-assessment to determine students who will benefit from explicit instruction* in academic vocabulary for a circle and parts.<br><br>• Students might be provided with transparent circles of various radii to help them discover a possible match.<br><br>• Students might use Patty Paper™ to | **Engage: Journal assignment – *Engage: Buried Treasure*** activity. Students will explain in their journal how they might determine the size of the circle from which a given chord and subtended arc were generated. They will then estimate the radius of the circle.<br>Additional Materials: Student Journals, ***Pottery Shard***<br><br>**Explore/Explain:** Students will investigate the Intersecting Chords Theorem via ***Segmented Circles 1.*** The instructor is encouraged to provide the name of the theorem to the students once the activity is complete. This is the basic activity that will be completed by Tables 1, 4, and 5.<br>Additional Materials: compass, ruler | Provide students with ***Segmented Circles 3***. Students who are ready for enrichment will complete a proof of the *Intersecting Chords Theorem* discovered in the ***Segmented Circles*** activity.<br>Additional Materials: ***Hint Board Templates*** |

| | | |
|---|---|---|
| determine the size of the circle.<br><br>**Explore/Explain:**<br><br>Groups may be assigned based on student level to allow more directed guidance where needed using a selection of the activities provided below.<br><br>• Students may be provided with circles that are pre-constructed on a grid in such a way that the lengths of the chords may be determined simply by counting the grid marks. One example is included in ***Segmented Circles 2***.<br><br>*Explicit Instruction includes teaching components such as:<br>• clear modeling of the solution specific to the problem;<br>• thinking the specific steps aloud during modeling;<br>• presenting multiple examples of the problem and applying the solution to the problems; and<br>• providing immediate corrective feedback to the students on their accuracy.<br><br>MSTAR Presenter's Guide pg 173 (2009) | **Formative Assessment:** Each student will receive a copy of ***Closing the Loop: Revisiting Buried Treasure***. Students will apply the result that was discovered in the ***Segmented Circles*** activity to discover the radius of the plate from the ***Buried Treasure*** activity.<br><br>Note:<br>Formative assessment items test concepts taught in the lesson and provide teachers valid information on whether students learned the concepts, principles and skills related to the lesson.<br><br>A transfer assessment question provides information on whether the students can take the concepts from the lesson and apply them in a novel situation. | |

## Engage: Buried Treasure

During an excavation of the Alamo battle site, archaeologists found the remains of some pottery. One of the pieces was from a plate they think was used by Davy Crockett. Archaeologists wish to create a replica of the plate from the details of the retrieved piece. The archaeologists know that the plate was originally in the shape of a disk. Suppose the piece of the broken plate is the same size and shape as the one drawn in the figure below. In your journal, explain how you might use this figure to estimate the size of the original circular plate. What do you think is the radius of the plate?



# Pottery Shard Template

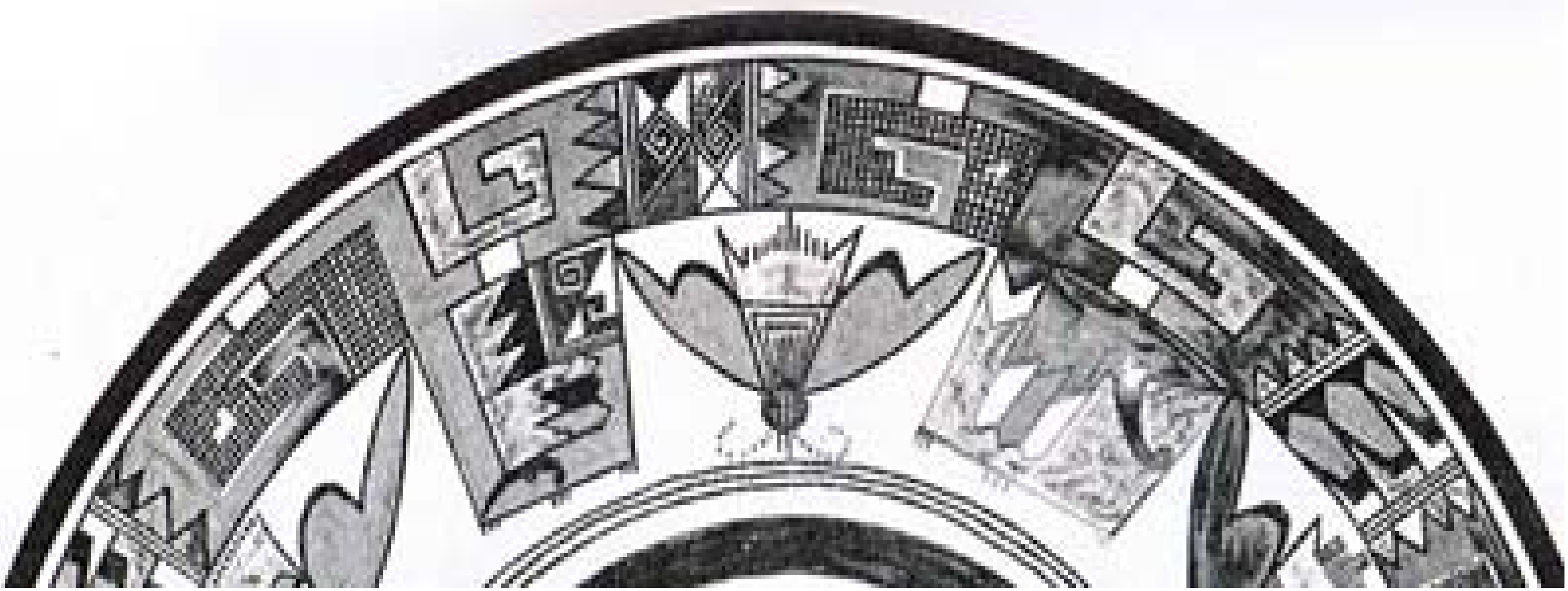


Table #1

Student

Characteristics

Table #2

Assessing

Prior

Knowledge

Table #3

Geometry Readiness,

Supporting Standards

And CCRS

Table #4

Journaling

Table #5

Tier 1

Instruction

**Table #6**

**Student Populations**

**Table #7**

**Assessing**

**Prior**

**Knowledge**

Table #8

Geometry Readiness,

Supporting Standards

And CCRS

Table #9

Journaling

Table #10

Tier 1

Instruction

| | |
|---|---|
| 1 | 2 |
| 3 | 4 |
| 5 | 6 |

# Center Folder Questions

**As you work through your center activity you will consider the following questions:**

1. **Is there evidence your "topic" is addressed in the activity?**
2. **If so, how?**
3. **If not what could be added to improve?**
4. **What is the advantage of having students work in groups on this activity?**

**As a group, create a poster to include:**

- **a summary of the activity that includes the overall objective of the lesson.**
- **an Illustration of how your topic was/could be addressed.**
- **a depiction of how the activity addressed the needs of each group member's assigned student characteristics.**

**As you work through your center activity you will consider the following questions:**

1. **Is there evidence your "topic" is addressed in the activity?**
2. **If so, how?**
3. **If not what could be added to improve?**
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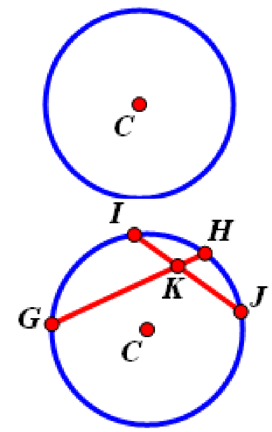
- **an Illustration of how your topic was/could be addressed.**
- **a depiction of how the activity addressed the needs of each group member's assigned student characteristics.**

# Segmented Circles 1: Table 1

## SEGMENTED CIRCLES




1. On a blank sheet of paper, use your compass to construct a circle with the radius of your choice. (Make sure your circle has a different radius than the other members of your group.)
2. Label the center of your circle as ***C***.
3. Construct any chord of your circle and label the endpoints of your chord as ***G*** and ***H***.
4. Construct a second chord of your circle that intersects the first. Label the endpoints of your second chord as ***I*** and ***J***.
5. Label the intersection point of the chords as ***K***.
6. Measure the lengths of the following line segments and record in the table below.

| LINE SEGMENT | LENGTH IN CENTIMETERS (to the nearest tenth of a cm) |
|---|---|
| $\overline{GK}$ | |
| $\overline{KH}$ | |
| $\overline{IK}$ | |
| $\overline{KJ}$ | |

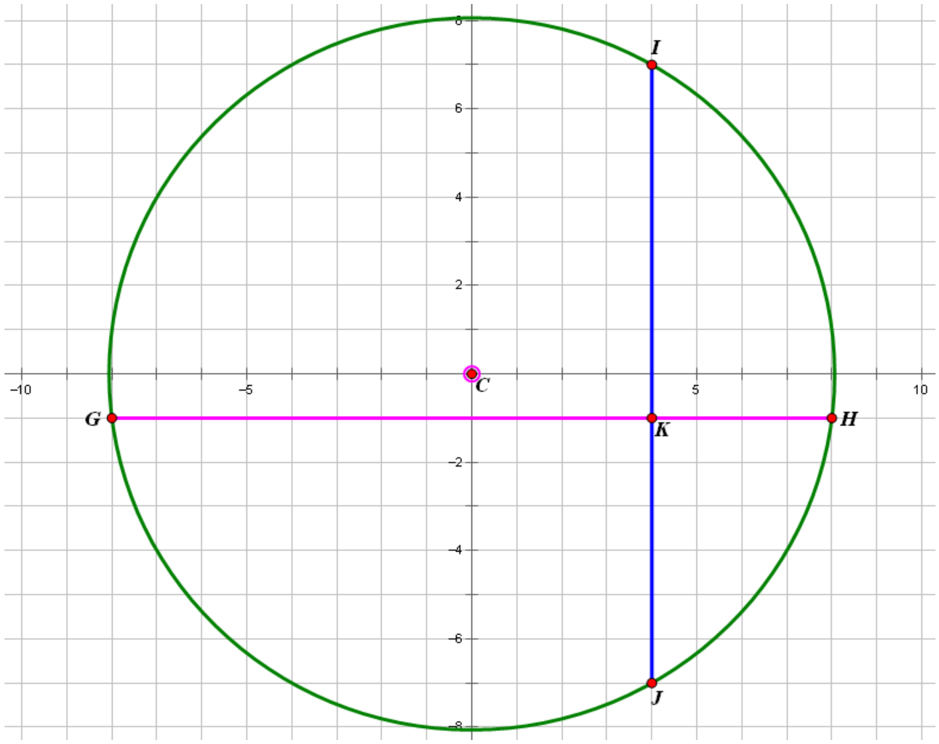
7. Calculate the product of the lengths of $\overline{GK}$ and $\overline{KH}$.
8. Calculate the product of the lengths of $\overline{IK}$ and $\overline{KJ}$.
9. What do you notice about the products that you calculated in the previous two steps?
10. Compare your results with those of your group mates.
11. Based on the observations of your group, make a conjecture about intersecting chords of a circle.

Could you make the same conjecture about rectangles? Illustrate why or why not.

# Segmented Circles 2: Table 2

**Small Group**

Below is a circle on a coordinate grid. The length of each square on the grid is one unit.




1. Trace chord GH with a yellow highlighter or colored pencil.
2. Trace chord IJ with a pink highlighter or colored pencil.
3. Where do chords GH and IJ intersect?
4. Find the lengths of the following line segments and record in the table below.

| LINE SEGMENT | LENGTH IN UNITS |
|---|---|
| $\overline{GK}$ | |
| $\overline{KH}$ | |
| $\overline{IK}$ | |
| $\overline{KJ}$ | |

5. Calculate the product of the lengths of $\overline{GK}$ and $\overline{KH}$.

6. Calculate the product of the lengths of $\overline{IK}$ and $\overline{KJ}$.

7. What do you notice about the products that you calculated in the previous two steps?

# Segmented Circles 2: Table 2

### Teacher's Scaffolding Prompts and Questions

**Segmented Circles 2, Small group-Teacher led activity**

Student Question 1

If a student is struggling to highlight chord GH, ask them to find point G and highlight it. Find point H and highlight it.  Now highlight the line that connects point G and point H.

Student Question 2

Same as above.

Student Question 3

What does the word intersect mean?
What is the name of the point located at the intersection, or where the lines cross?

Student Question 4

Trace line segment GK using your pencil.
How can you find the length of line segment GK? (By count the squares.)
Repeat prompts for the remaining line segments.

It is important for students to realize line segment GK and line segment KH make up chord GH and line segment IK and KJ make up chord IJ.

When line segments GK and line segment KH are added together, what line segment do they make?
When line segments IK and line segment KJ are added together, what line segment do they make?

Student Question 5

What math operation does the word product indicate?
What numbers will you multiple?

Student Question 6

Same as above.

Student Question 7

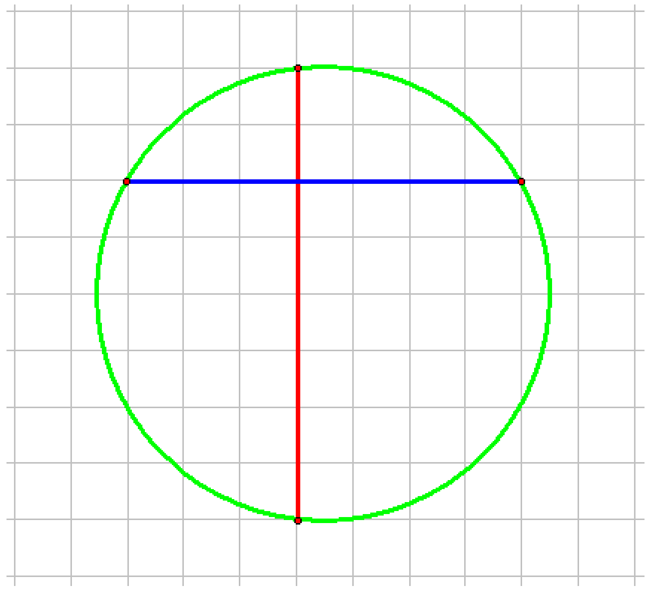
If students did not get the same product on number 5 and 6, assist them in recalculating the products.

After students have successfully completed Segmented Circles 2: Small Group, distribute a different circle from **Segmented Circles 2: Circle 1, Segmented Circles 2: Circle 2, Segmented Circles 2: Circle 3, Segmented Circles 2: Circle 4 or Segmented Circles 2: Circle 5** to each student. Monitor and assist using the same scaffolding questions above. When students have completed this page, lead a discussion in order for students to conclude the products of the line segments of their chords were equal.

# Segmented Circles 2: Table 2

## Circle 1

Below is a circle on a coordinate grid.  The length of each square is one unit.



1. Label the endpoints of one chord as ***B*** and **C**.
2. Label the endpoints of one chord as ***D*** and **F**.
3. Label the intersection point of the chords as **G**.
4. Measure the lengths of the following line segments and record in the table below.

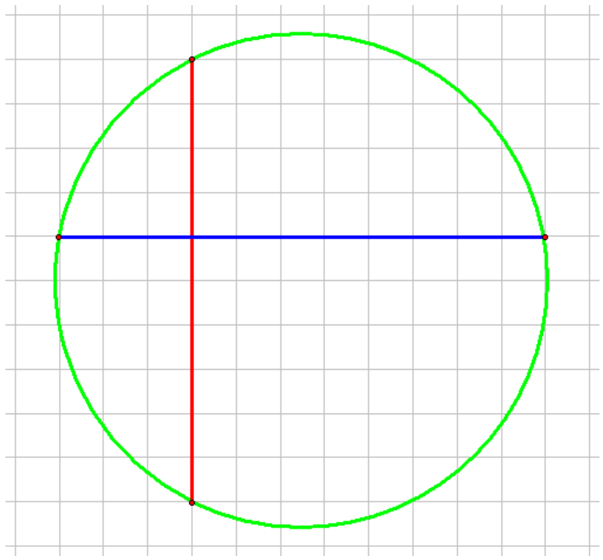
| LINE SEGMENT | LENGTH IN UNITS |
|---|---|
| $\overline{BG}$ | |
| $\overline{GC}$ | |
| $\overline{DG}$ | |
| $\overline{GF}$ | |

5. Calculate the product of the lengths of $\overline{BG}$ and $\overline{GC}$.
6. Calculate the product of the lengths of $\overline{DG}$ and $\overline{GF}$.
7. What do you notice about the products that you calculated in the previous two steps?
8. Compare your results with those of your group members.
9. Based on the observations of your group, make a conjecture about intersecting chords of a circle.

# Segmented Circles 2: Table 2

**Circle 2**

Below is a circle on a coordinate grid.  The length of each square is one unit.



1. Label the endpoints of one chord as ***B*** and **C**.
2. Label the endpoints of one chord as ***D*** and **F**.
3. Label the intersection point of the chords as **G**.
4. Measure the lengths of the following line segments and record in the table below.

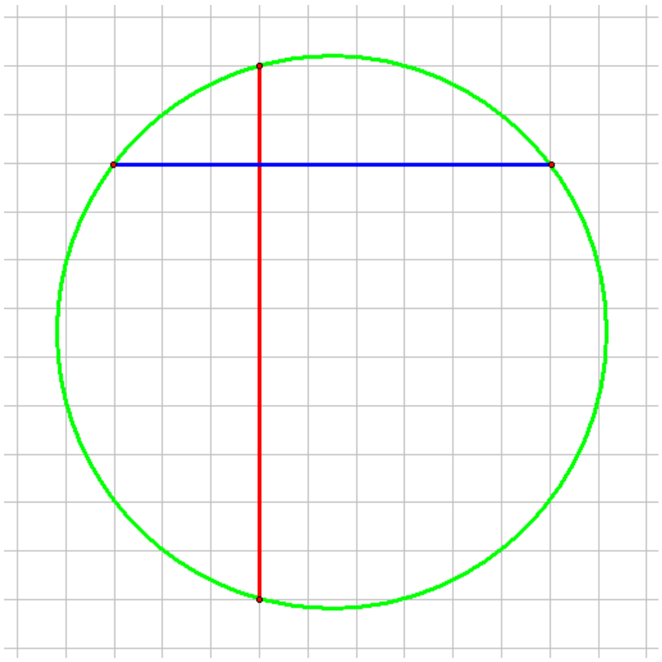
| LINE SEGMENT | LENGTH IN UNITS |
|---|---|
| $\overline{BG}$ | |
| $\overline{GC}$ | |
| $\overline{DG}$ | |
| $\overline{GF}$ | |

5. Calculate the product of the lengths of $\overline{BG}$ and $\overline{GC}$.
6. Calculate the product of the lengths of $\overline{DG}$ and $\overline{GF}$.
7. What do you notice about the products that you calculated in the previous two steps?
8. Compare your results with those of your group members.
9. Based on the observations of your group, make a conjecture about intersecting chords of a circle.

## Segmented Circles 2: Table 2

### Circle 3

Below is a circle on a coordinate grid. The length of each square is one unit.



1. Label the endpoints of one chord as ***B*** and **C**.
2. Label the endpoints of one chord as ***D*** and **F**.
3. Label the intersection point of the chords as **G**.
4. Measure the lengths of the following line segments and record in the table below.

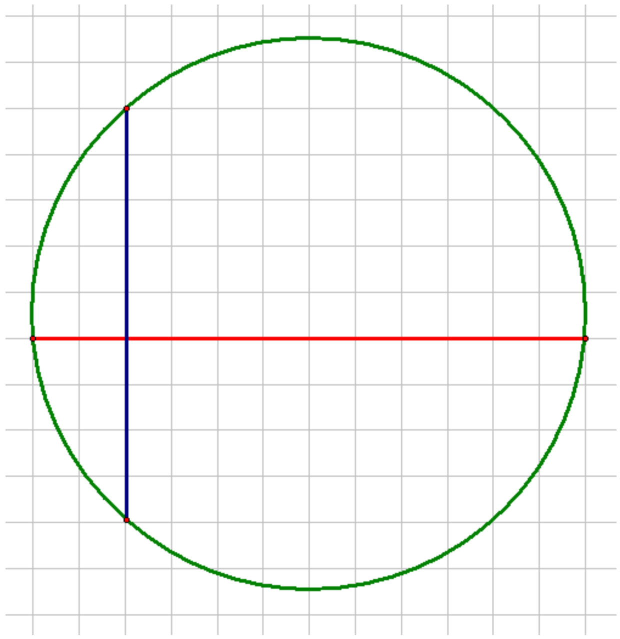
| LINE SEGMENT | LENGTH IN UNITS |
|---|---|
| $\overline{BG}$ | |
| $\overline{GC}$ | |
| $\overline{DG}$ | |
| $\overline{GF}$ | |

5. Calculate the product of the lengths of $\overline{BG}$ and $\overline{GC}$.
6. Calculate the product of the lengths of $\overline{DG}$ and $\overline{GF}$.
7. What do you notice about the products that you calculated in the previous two steps?
8. Compare your results with those of your group members.
9. Based on the observations of your group, make a conjecture about intersecting chords of a circle.

# Segmented Circles 2: Table 2

### Circle 4

Below is a circle on a coordinate grid.  The length of each square is one unit.



1. Label the endpoints of one chord as ***B*** and **C**.
2. Label the endpoints of one chord as ***D*** and **F**.
3. Label the intersection point of the chords as **G**.
4. Measure the lengths of the following line segments and record in the table below.

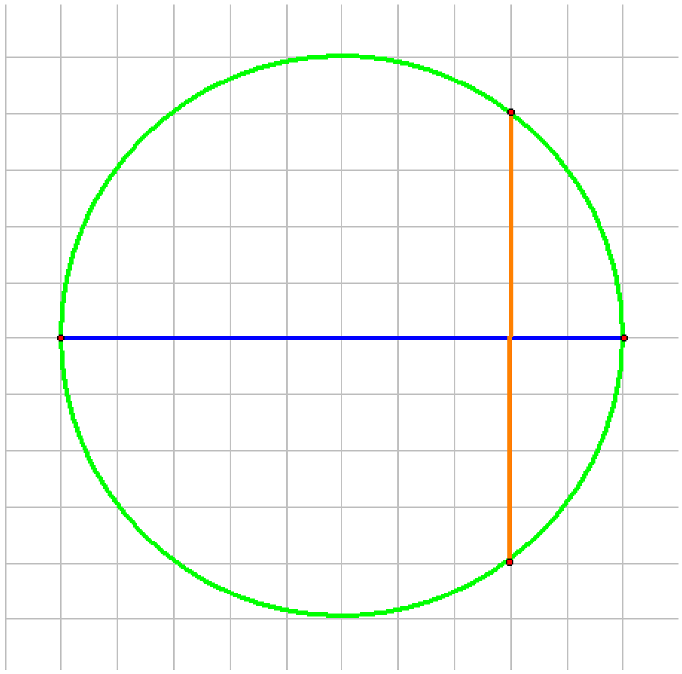
| LINE SEGMENT | LENGTH IN UNITS |
|---|---|
| $\overline{BG}$ | |
| $\overline{GC}$ | |
| $\overline{DG}$ | |
| $\overline{GF}$ | |

5. Calculate the product of the lengths of $\overline{BG}$ and $\overline{GC}$.
6. Calculate the product of the lengths of $\overline{DG}$ and $\overline{GF}$.
7. What do you notice about the products that you calculated in the previous two steps?
8. Compare your results with those of your group members.
9. Based on the observations of your group, make a conjecture about intersecting chords of a circle.

## Segmented Circles 2: Table 2

### Circle 5

Below is a circle on a coordinate grid. The length of each square is one unit.



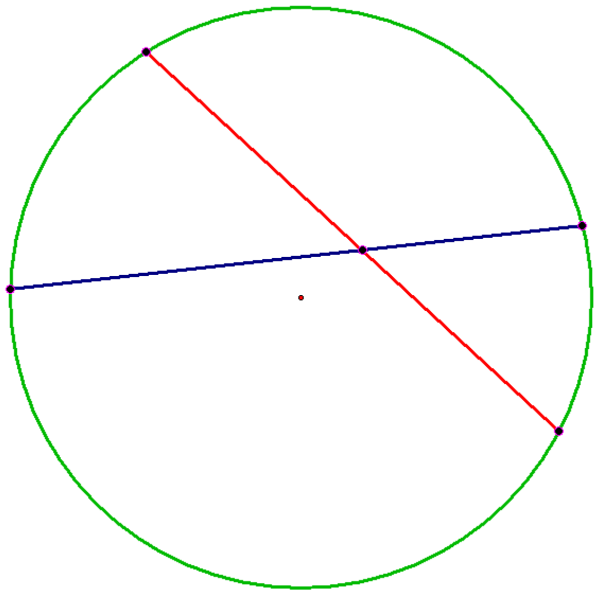
1. Label the endpoints of one chord as ***B*** and **C**.
2. Label the endpoints of one chord as ***D*** and **F**.
3. Label the intersection point of the chords as **G**.
4. Measure the lengths of the following line segments and record in the table below.

| LINE SEGMENT | LENGTH IN UNITS |
|---|---|
| $\overline{BG}$ | |
| $\overline{GC}$ | |
| $\overline{DG}$ | |
| $\overline{GF}$ | |

5. Calculate the product of the lengths of $\overline{BG}$ and $\overline{GC}$.
6. Calculate the product of the lengths of $\overline{DG}$ and $\overline{GF}$.
7. What do you notice about the products that you calculated in the previous two steps?
8. Compare your results with those of your group members.
9. Based on the observations of your group, make a conjecture about intersecting chords of a circle.

# Segmented Circles 2: Table 2

## One More Circle



1. Label the center of your circle as ***C***.
2. Label the endpoints of a chord as **R** and ***T***.
3. Label the endpoints of the second chord as ***W*** and ***X***.
4. Label the intersection point of the chords as ***V***.
5. Measure the lengths of the following line segments and record in the table below.

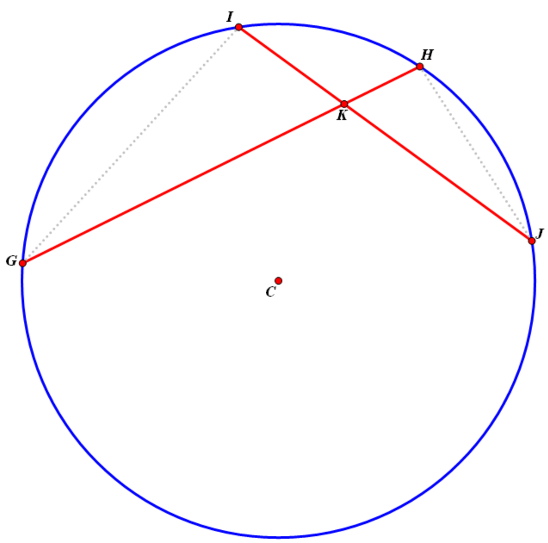
| LINE SEGMENT | LENGTH IN CENTIMETERS<br><br>( to nearest tenth of a CM) |
|---|---|
| $\overline{RV}$ | |
| $\overline{VT}$ | |
| $\overline{WV}$ | |
| $\overline{VX}$ | |

6. Calculate the product of the lengths of $\overline{RV}$ and $\overline{VT}$.
7. Calculate the product of the lengths of $\overline{WV}$ and $\overline{VX}$.
8. What do you notice about the products that you calculated in the previous two steps?
9. Compare your results with those of your group members.
10. Based on the observations of your group, make a conjecture about intersecting chords of a circle. (Write on the back of the paper if necessary.)

# Segmented Circles 3: Table 3

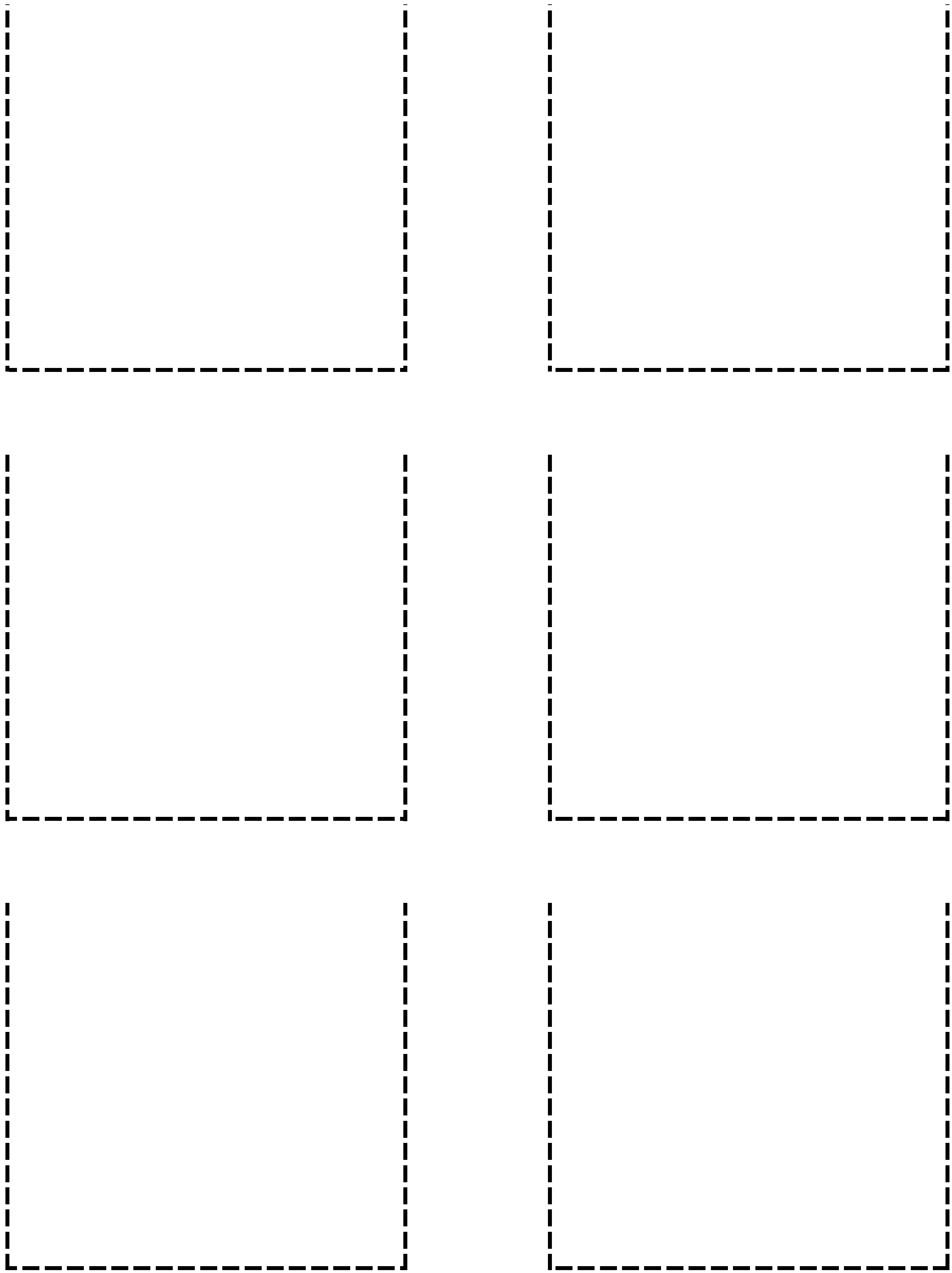
### Proof of the Intersecting Chord Theorem

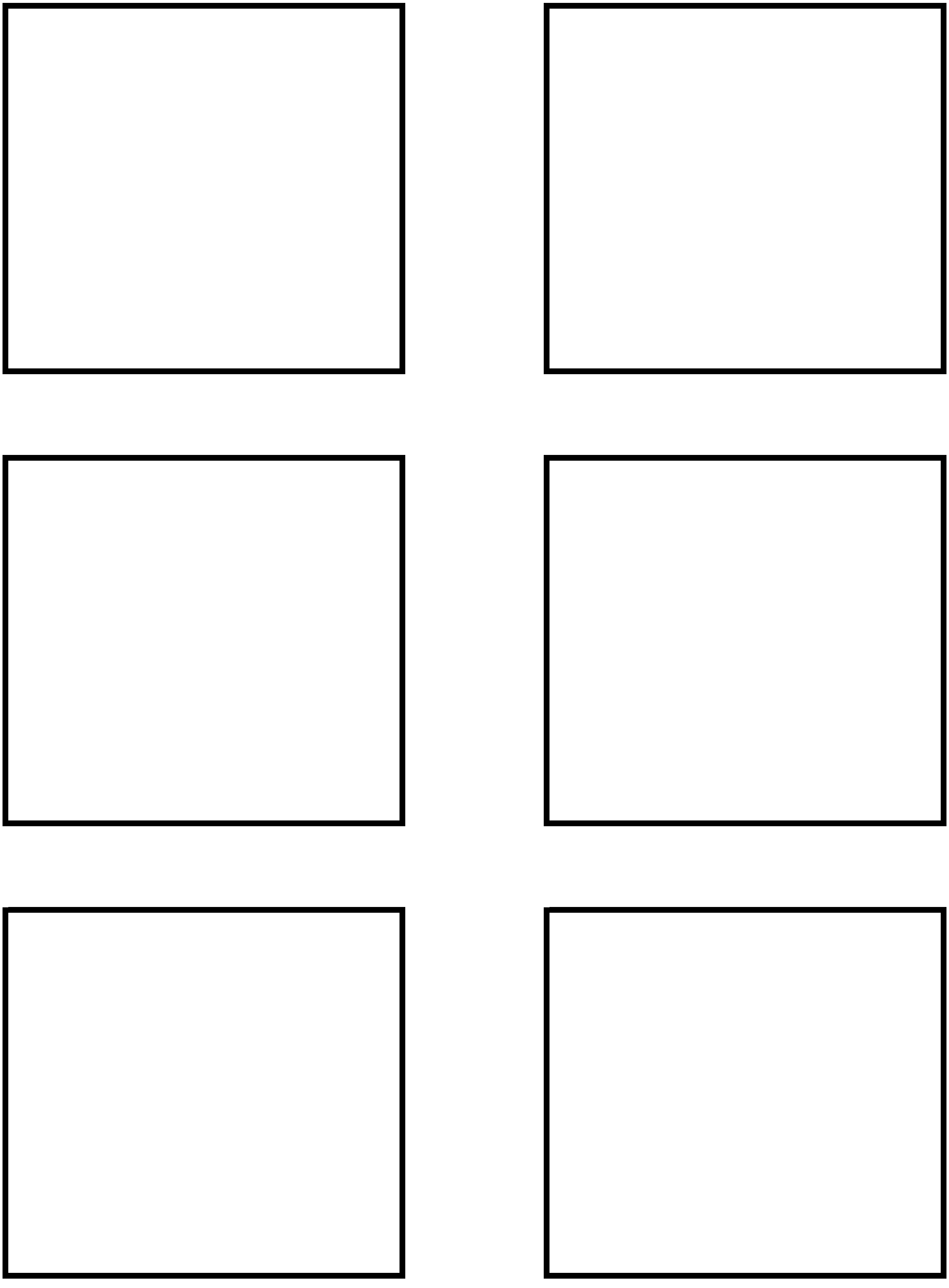
Prove the ***Intersecting Chords Theorem:*** *Given an interior point K of any circle and two chords* $\overline{GH}$ *and* $\overline{IJ}$ *of the circle that intersect at the point K. Then the product of the lengths of* $\overline{GK}$ *and* $\overline{KH}$ *is equal to the product of the lengths of* $\overline{IK}$ *and* $\overline{KJ}$.




Some may need help or "hints" to complete the proof of the Intersecting Chord Theorem. You are to create "hints" for the "Hint Board". If someone decides they need to use the "Hint Board", they will have to *buy* a hint. It is up to you to also decide on the point value for the hints.

- Write one hint per square
- Decide the point value for each hint
- Using the template create the "Hint Board"





**Segmented Circles 4: Table 4**

## Nspire Activity

Follow the steps on the “Intersecting Chords” video to complete the construction of a circle and two intersecting chords.

Use the “drag” capabilities of the Nspire to change the lengths of the chords.

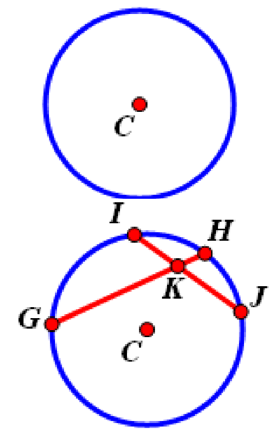
Record data in the table below.

| | AB | BD | AB·BD | EB | BC | EB·BC |
|---|---|---|---|---|---|---|
| Sample 1 | | | | | | |
| Sample 2 | | | | | | |
| Sample 3 | | | | | | |
| Sample 4 | | | | | | |
| Sample 5 | | | | | | |
| Sample 6 | | | | | | |

Make a conjecture about the intersecting chords of a circle.

# Segmented Circles 5: Table 5

## SEGMENTED CIRCLES




1. On a blank sheet of paper, use your compass to construct a circle with the radius of your choice. (Make sure your circle has a different radius that the other members of your group.)
2. Label the center of your circle as ***C***.
3. Construct any chord of your circle and label the endpoints of your chord as ***G*** and ***H***.
4. Construct a second chord of your circle that intersects the first. Label the endpoints of your second chord as ***I*** and ***J***.
5. Label the intersection point of the chords as ***K***.
6. Measure the lengths of the following line segments and record in the table below.

| LINE SEGMENT | LENGTH IN CENTIMETERS (to the nearest tenth of a cm) |
|---|---|
| $\overline{GK}$ | |
| $\overline{KH}$ | |
| $\overline{IK}$ | |
| $\overline{KJ}$ | |

7. Calculate the product of the lengths of $\overline{GK}$ and $\overline{KH}$.
8. Calculate the product of the lengths of $\overline{IK}$ and $\overline{KJ}$.
9. What do you notice about the products that you calculated in the previous two steps?
10. Compare your results with those of your group mates.
11. Based on the observations of your group, make a conjecture about intersecting chords of a circle.

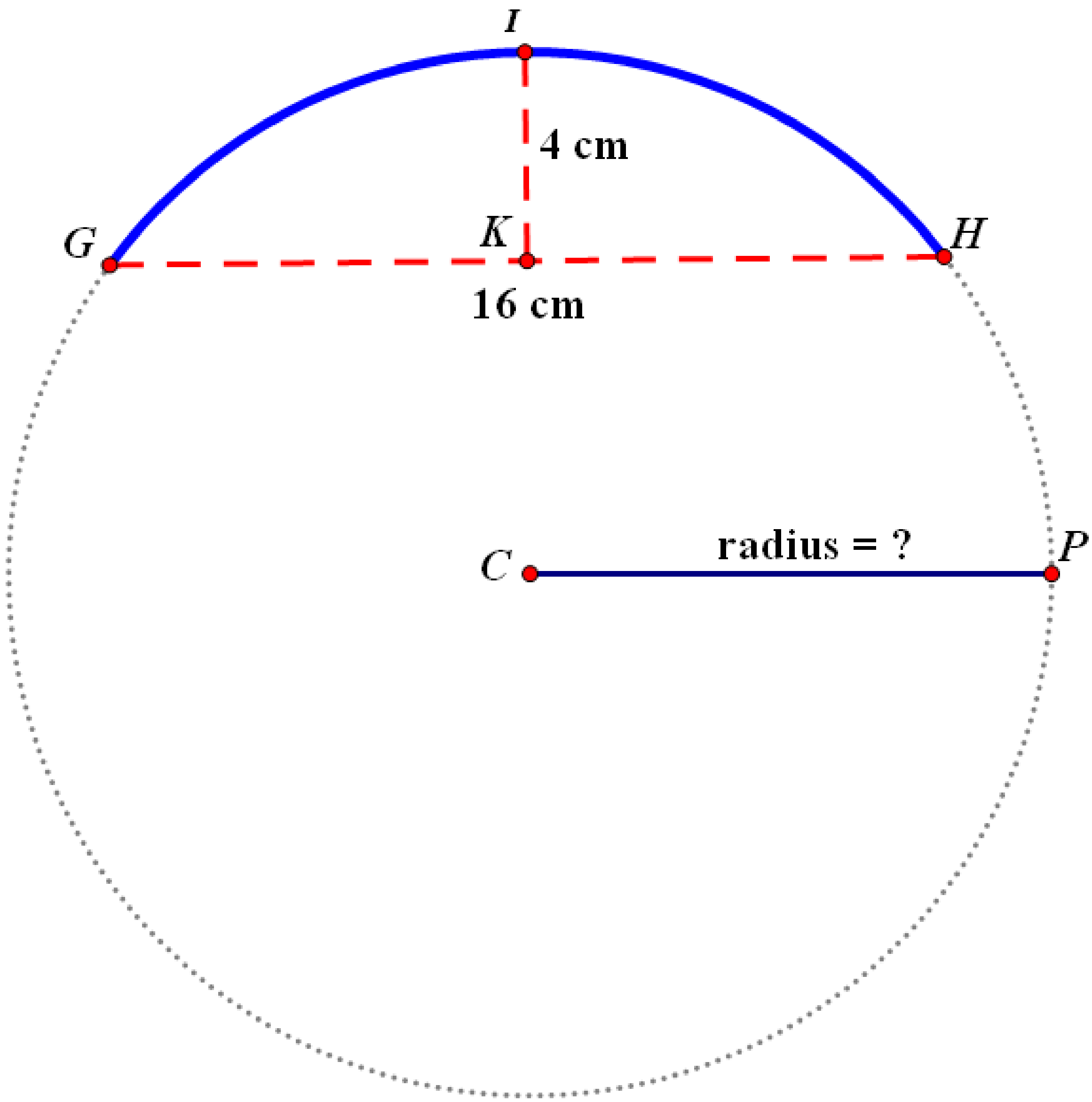
Video Piece: Your friend Jessie was absent from class today, using one of the constructions from your group and the flip video camera, explain to Jessie your conjecture about intersecting chords of a circle.

Video requirements:

- Video should last no longer than 3 minutes
- No faces shown in the video recording

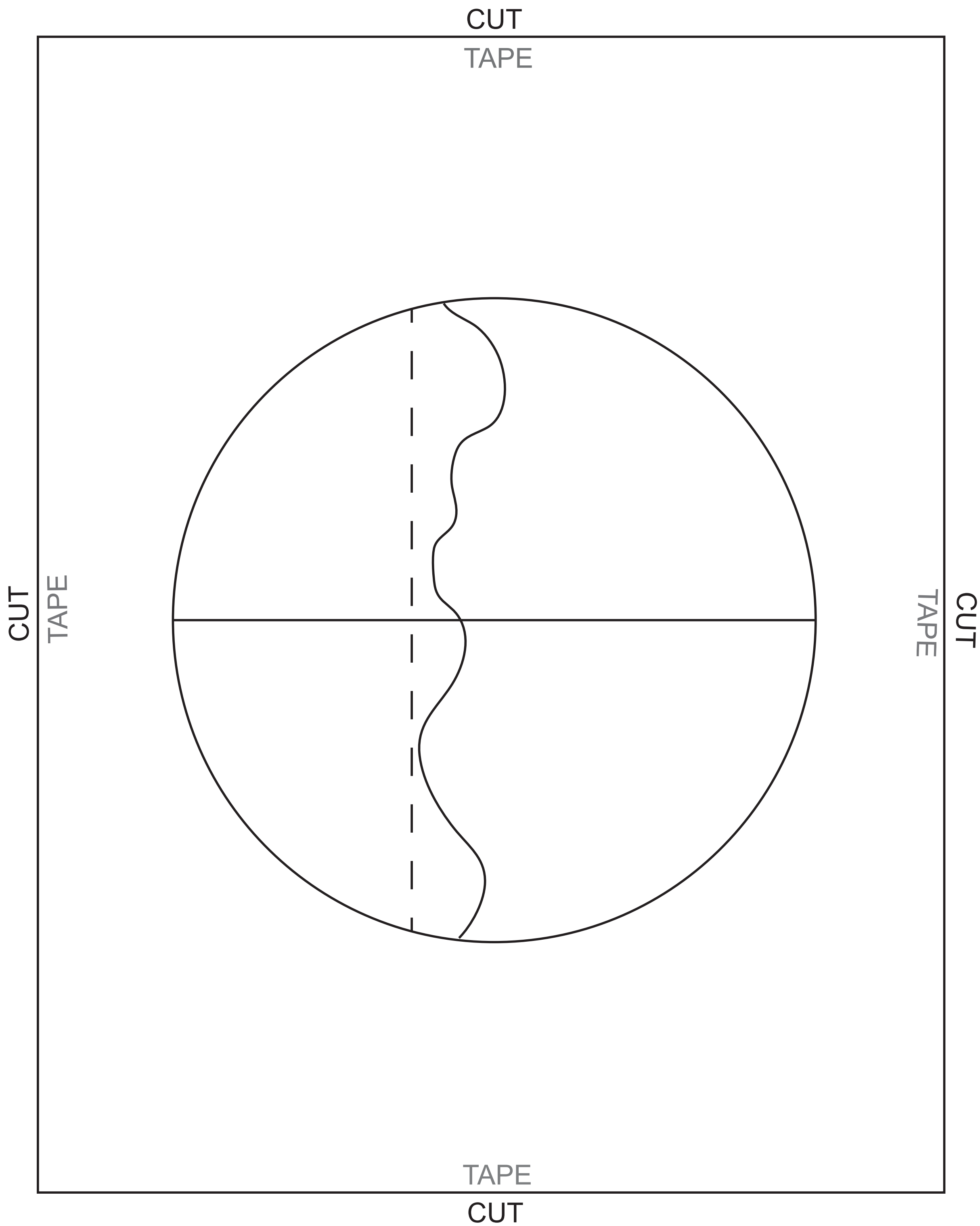
## Closing the Loop: Revisiting Buried Treasure

In the ***Buried Treasure*** activity, you estimated the size of the broken circular plate based on the size of the piece the archaeologists found. In the image below, the piece and its measurements have been sketched onto a circle. Suppose segment ***IK*** is a perpendicular bisector of ***GH.*** Use the ***Intersecting Chords Theorem*** to determine the radius of the circle.



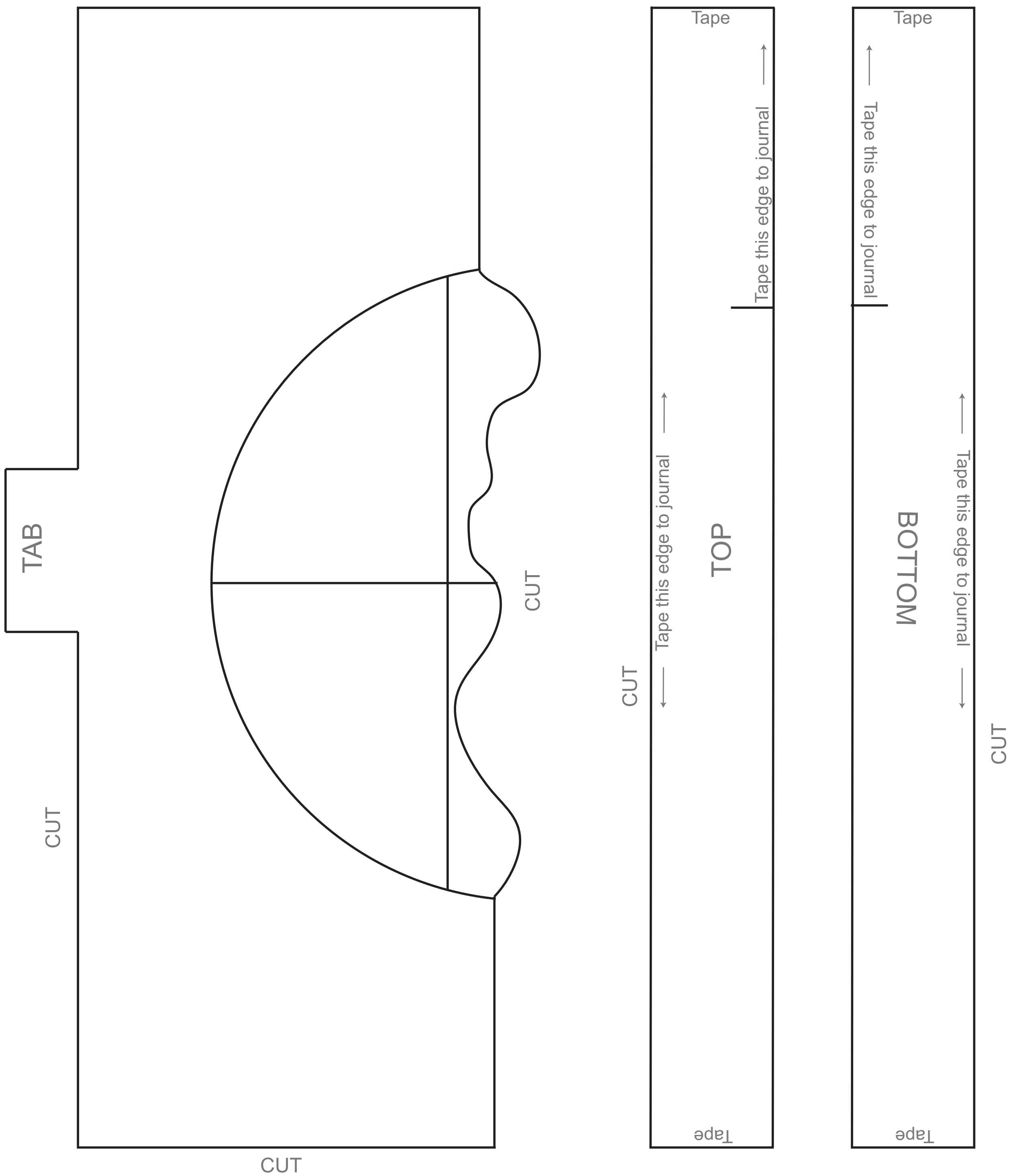
# Segmented Circles Journal Activity

Cut along outside edge, tape into journal



# Segmented Circles Journal Activity

Cut along outside edge, tape into journal



# Right is Special

# Geometry EOC Success
## Lesson Plan Summary: *Right is Special*

**Topic:** Develop relationships between the sides of special right triangles and connect to trigonometric ratios

**CCRS:** In this lesson, the student will:

- make and validate geometric conjectures.
- recognize and apply right triangle relationships to trigonometric ratios.
- use dilations to investigate trigonometric ratios.
- use mathematical language to represent and communicate concepts.

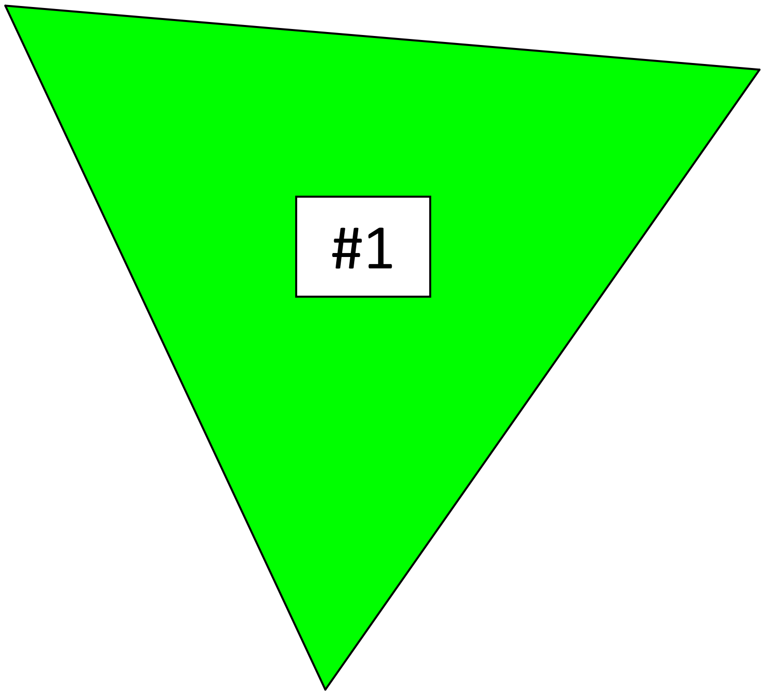
| | |
|---|---|
| **Content Objective:** The student uses special right triangles to make, test, and justify conjectures. | **Language Objective:**<br>C3(D) The student is expected to speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency. |
| **Vocabulary:** hypotenuse, legs of right triangles, concentric circles, trigonometric ratios of sine, cosine and tangent | **Prior Knowledge:** Students are expected to identify similar triangles and write proportional relationships. The Pythagorean Theorem is applied in order to find the third side of a right triangle. |

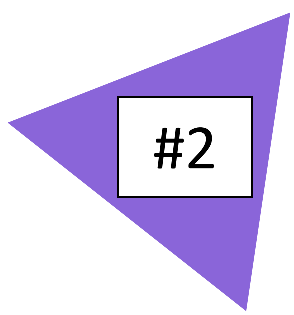
| RtI Tier I<br>Differentiation Activity | Instructional Phase | Enrichment Differentiation Activity |
|---|---|---|
| ***Mini-teach**: Students have been working with proportional reasoning since 6th grade and similar triangles since 7th grade; however, Pythagorean Theorem is introduced in 8th grade and students may not have had experience with this for the past year. Teachers should provide an *explicit review of this topic prior to the engage activity. This instruction should include both instruction on how to find *c* given *a* and *b* as well as how to find *b* given *a* and *c*.<br><br>**Engage:** | **Engage: Journal Assignment**<br>Students will be given an equilateral triangle and ask to record as many relationships that may be true. There are 5 triangles available on ***Engage: Equilateral Cutouts***. If students are working in groups, each student should have a different triangle.<br><br>**Explore:**<br>For Right is Special 1:***Triangles on a Grid***, students will place 30-60-90 triangles on the grid paper and make conjectures about the relationships between the triangles and the ratio of corresponding sides of the triangles.<br>Additional Materials: compasses, straight edges<br><br>Right is Special 2: ***More Triangles on a Grid*** shows a graph with five 30-60-90 triangles. Students will be asked about what | If students have mastered the instructional phase, give students Right is Special 6: ***What Do You Think and Why?*** Given a right triangle other than a 30-60-90 triangle, students will make conjectures about the relative size of trigonometric ratios as compared to the trigonometric ratios of the 30-60-90 triangles. |

<table>
<tr>
<td rowspan="3">• Based on student responses in the Equilateral Cut-Outs activity, a mini-teach is appropriate on the identified areas of need.<br><br>• Right is Special 4: Vocabulary Pictures provides students will see relationships in pictures. After the pictures true/false statements will be given.<br><br>Explore/Explain:<br>• In Right is Special 5: Seeing Triangles on a Grid students will be given first the triangle, then the circle and asked to find relationships. The students will find lengths of sides and trigonometric ratios.<br>Additional Materials: protractors<br><br>*Explicit Instruction includes teaching components such as:<br>• clear modeling of the solution specific to the problem<br>• thinking the specific steps aloud during modeling<br>• presenting multiple examples of the problem and applying the solution to the problems, and<br>• providing immediate corrective feedback to the students on their accuracy.<br>MSTAR Presenter's Guide pg 173 (2009)</td>
<td>relationships exist and to justify their answers.</td>
<td></td>
</tr>
<tr>
<td>Explain:<br>Teacher will do a mini-teach on the definitions of sine, cosine and tangent trigonometric ratios.  Students will complete a chart on Right is Special 3: Sine, Cosine, Tangent as a graphic organizer to make and confirm conjectures.</td>
<td rowspan="2"></td>
</tr>
<tr>
<td>Formative Assessment:<br>Students will be asked to complete three sentence stems to explain how to find the measure of unknown sides of the 30-60-90 special right triangles and how to find the value of trigonometric ratios in Closing the Loop: Knowing Right.  In addition, students will be asked to answer a question about the relationship between two 30-60-90 triangles.<br><br>Note:<br>Formative assessment items test concepts taught in the lesson and provide teachers valid information on whether students learned the concepts, principles and skills related to the lesson.<br><br>A transfer assessment question provides information on whether the students can take the concepts from the lesson and apply them in a novel situation.<br><br>Right is Special 6 could be used as a transfer assessment for this lesson.  Many times state assessments require transfer of knowledge, therefore; both types of questions should be used.  It is necessary to remember transfer items require students have a wide range of examples; these provide the background knowledge essential for transfer of information.</td>
</tr>
</table>

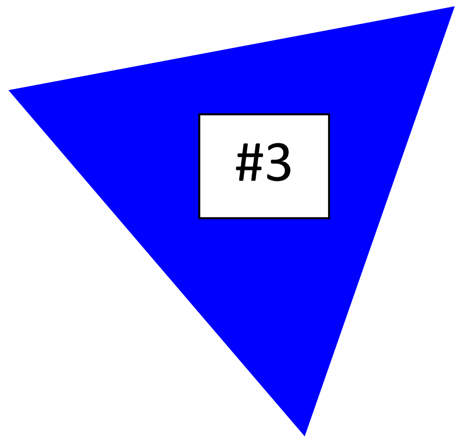
# Engage: Equilateral Cut-Outs

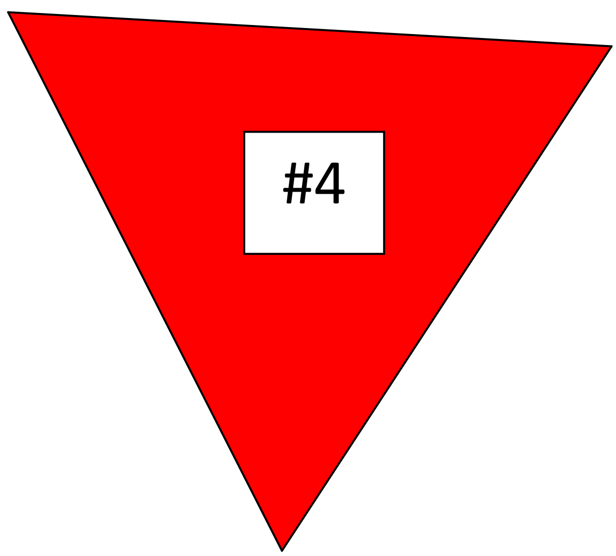
Cut out one of the equilateral triangles. Each member in your group should have a different triangle. Write a journal entry that includes as many relationships and concepts that might apply to the triangle. Folding the triangle in order to compare angles and segments may help.



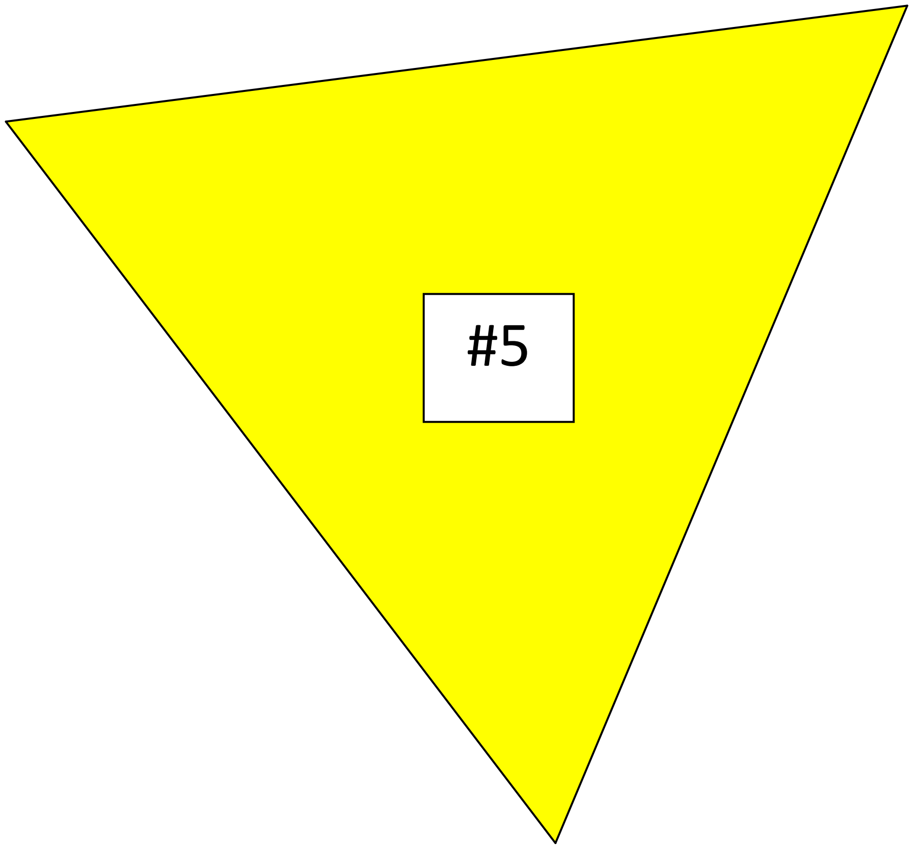



## Engage: Equilateral Cut-Outs



## Engage: Equilateral Cut-Outs

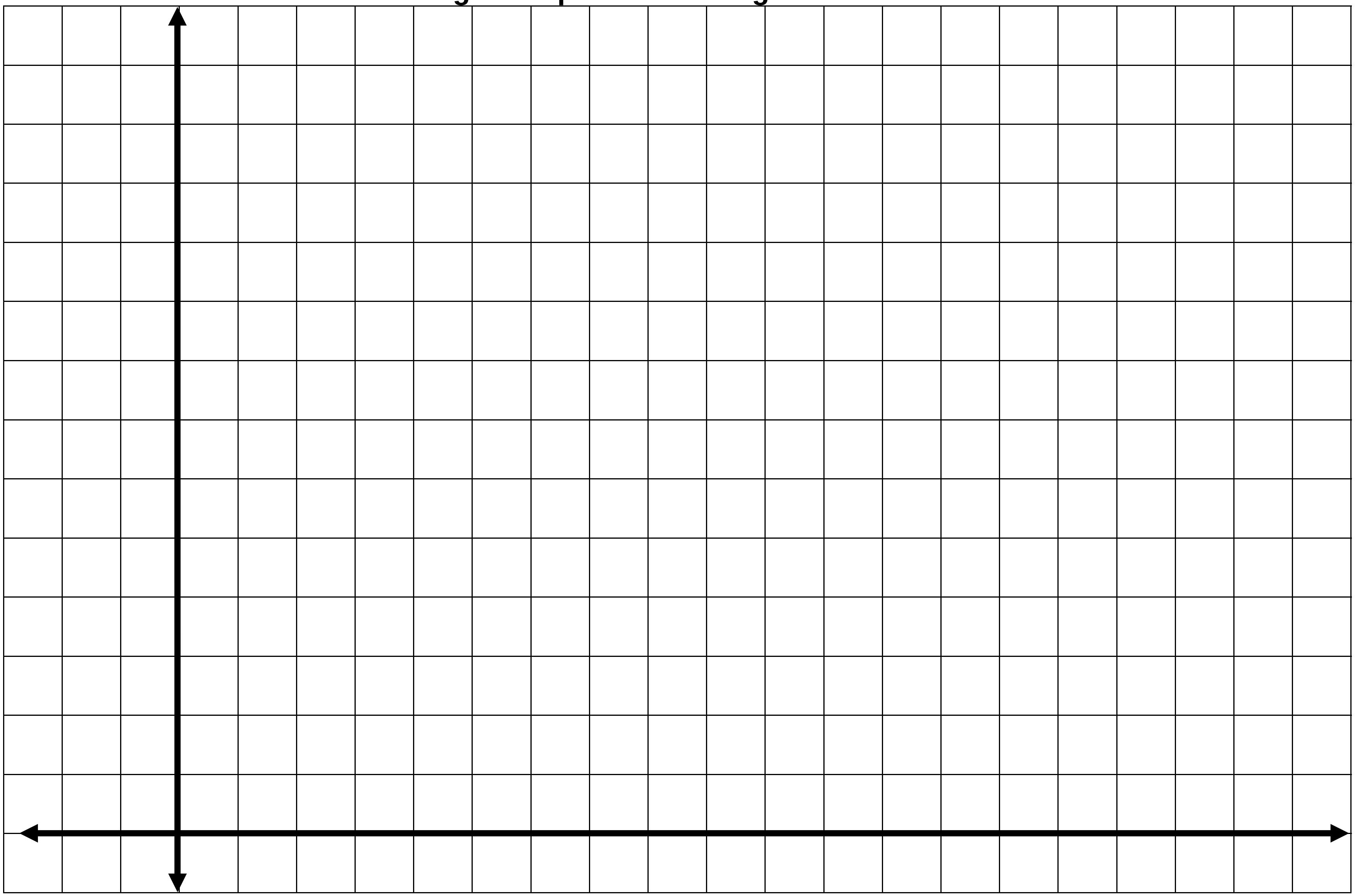


# Right is Special 1: Triangles on a Grid

Each student in your group should have a different equilateral triangle. Complete the following steps:

- Using a compass, set the compass points equal to the length of the side of the equilateral triangle.
- Draw a quarter of a circle on the grid paper with the center at the origin and the radius equal to the length of a side of the equilateral triangle.
- Fold the triangle in half in order to form a right triangle. Write the measure of each angle on this right triangle.
- Calculate the length of each side of the right triangle. The Pythagorean Theorem will be needed.  Leave the answer in simplified radical form. Write the measure on each side of the triangle.
- Place the vertex of the 30º angle at the origin and the longer leg along *x*-axis.
- Write the ordered pair at the point where the triangle intersects the circle.
    - In a right triangle, the ratio of the length of the leg opposite an angle divided by the length of the hypotenuse is called the sine of an angle. What is the ratio of sin 30º? (The abbreviation of sine is sin.)
    - In a right triangle, the ratio of the length of the leg adjacent to an angle divided by the length of the hypotenuse is called the cosine of an angle. What is the ratio of cos 30º? (The abbreviation of cosine is cos.)
    - In a right triangle, the ratio of the length of the leg opposite of an angle divided by the length of the leg adjacent to an angle is called the tangent of an angle. What is the ratio of tan 30º? (The abbreviation of tangent is tan.)

# Right is Special 1: Triangles on a Grid



# Right is Special 2: More Triangles on a Grid

Five 30-60-90 triangles are placed on graph paper. Use the picture below to answer the following questions:


1. Is there a relationship between the triangles? If so, what relationships exist? Justify your answer.

2. What relationships occur between two 30-60-90 triangles if each has a short leg with the same measure?

## Right is Special 3: Sine, Cosine, Tangent

1. Write the trigonometric ratios for the angle $\theta$ :

$\sin\,\theta =$ ___________ $\cos\,\theta =$ ___________ $\tan\,\theta =$ __________

2. Complete the table for the $30^{o}$ angle. Simplify each ratio.

| # of degrees | Trig ratio | Ratio for Triangle #1 | Ratio of Triangle #2 | Ratio of Triangle #3 | Ratio of Triangle #4 | Ratio of Triangle #5 |
|---|---|---|---|---|---|---|
| $30^{o}$ | $\sin 30^{o}$ | | | | | |
| $30^{o}$ | $\cos 30^{o}$ | | | | | |
| $30^{o}$ | $\tan 30^{o}$ | | | | | |

3. Use the table to complete the following:
    - A quick review of the table shows that each trigonometric ratio is equal no matter which triangle is used. Write a statement to explain why the size of the triangles does not affect the trigonometric ratio.
    - Write a statement explaining how the the coordinates written on the grid to the values in the table.

## Right is Special 3: Sine, Cosine, Tangent

4. Place each triangle on the grid paper in such a way so the 60º angle of the triangle is at the vertex and the short leg lies along the *x*-axis. Compare the coordinates of the point where the triangle intersects the circle to values in the table. How do the coordinates of the intersection point correspond to the ratios in the table below?

| # of degrees | Trig ratio | Ratio for Triangle #1 | Ratio of Triangle #2 | Ratio of Triangle #3 | Ratio of Triangle #4 | Ratio of Triangle #5 |
|---|---|---|---|---|---|---|
| 60º | *sin* 60º | | | | | |
| 60º | *cos* 60º | | | | | |
| 60º | *tan* 60º | | | | | |

- A quick review of the table shows that each trigonometric ratio is equal no matter which triangle is used. Write a statement to explain why the size of the triangles does not affect the trigonometric ratio.

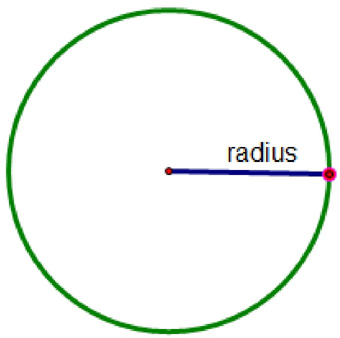
- Write a statement comparing the coordinates written on the grid to the values in the table.

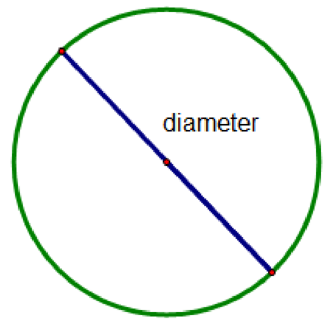
5. Find the length of the unknown legs if the hypotenuse is 20 units.

**Right is Special 4: Vocabulary Pictures**

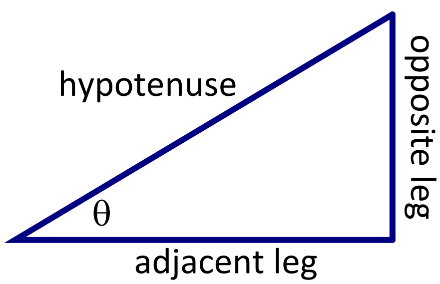
**A picture is worth a 1000 words.**

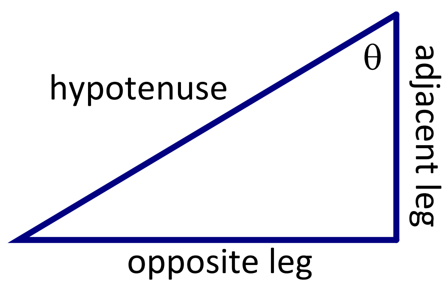
Assume the triangles have one right angle.

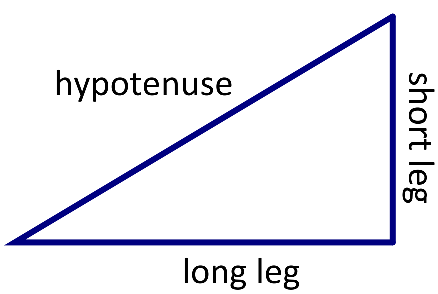


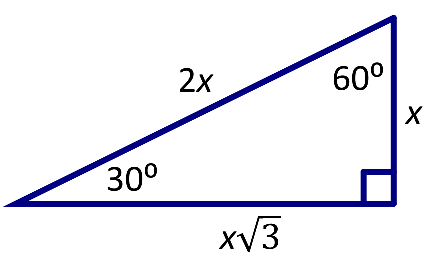

**What's the difference?**

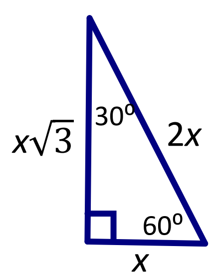



**Where are the sides in relationship to the angles?**



## Right is Special 4: Vocabulary Pictures

Are these statements true or false?

1. On any triangle the short leg is opposite the smallest angle.

2. On a right triangle the hypotenuse is opposite the right angle.

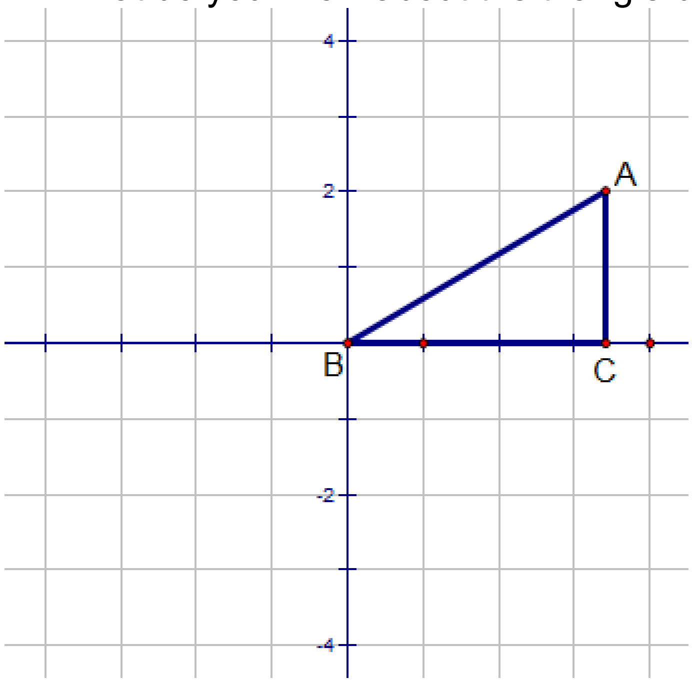
3. On a 30-60-90 triangle adjacent to the 60º angle is the longer leg.

4. On a 30-60-90 triangle the longer side and the hypotenuse form the 30º angle.

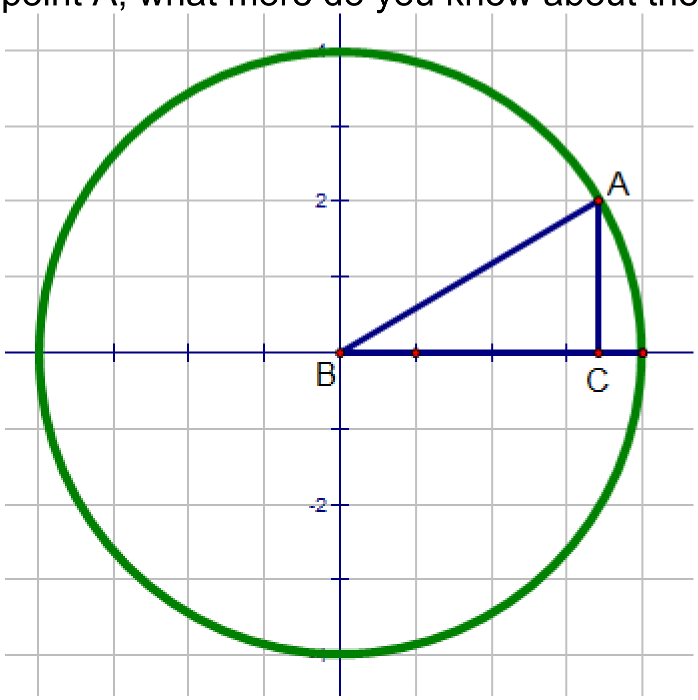
5. On a 30-60-90 triangle the longer leg is the product of the shorter leg and $\sqrt{3}$.

# Right is Special 5: Seeing Triangles on the Grid

1. What do you know about the triangle below?




2. After drawing the circle with the center at the origin and passing through point A, what more do you know about the triangle? Justify your answer.



3. You should have enough information to find the measure of all three sides of triangle. Use Pythagorean Theorem to find the third side of the triangle. Leave the measure in simple radical form. Label the sides on the graph above.

4. What are the coordinates of the point *A*?

5. With a protractor measure the angle *ABC*. Round to the nearest 10 degrees. $\angle ABC$ = ______________.

6. Without measuring angle *BAC* with a protractor, what is the measure? Justify your answer.

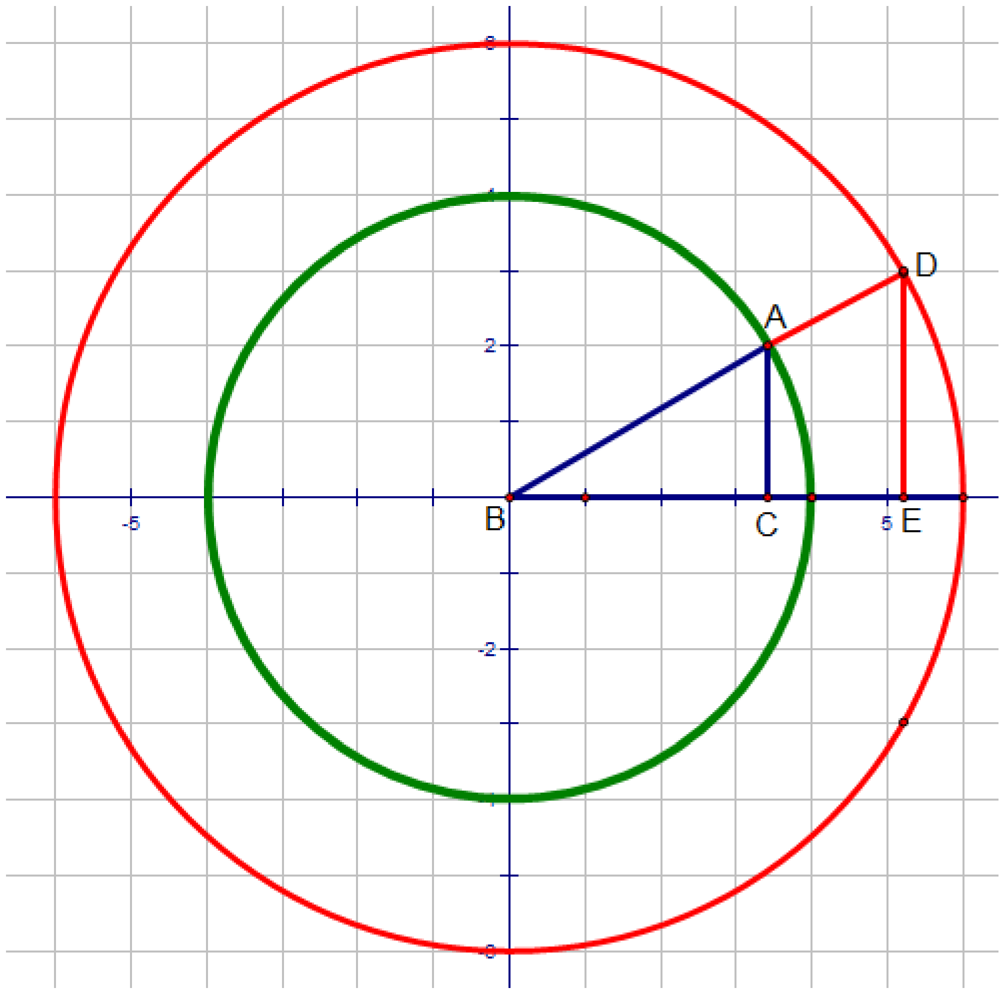
7. If the sine ratio is the ratio of the length of the opposite leg and the length of the hypotenuse, what does sin 30º equal?

8. If the cosine ratio is the ratio of the length of the adjacent leg and the length of the hypotenuse, what does cos 30º equal?

9. If the tangent ratio is the ratio of the length of the opposite leg and the length of the adjacent leg, what does tan 30º equal?

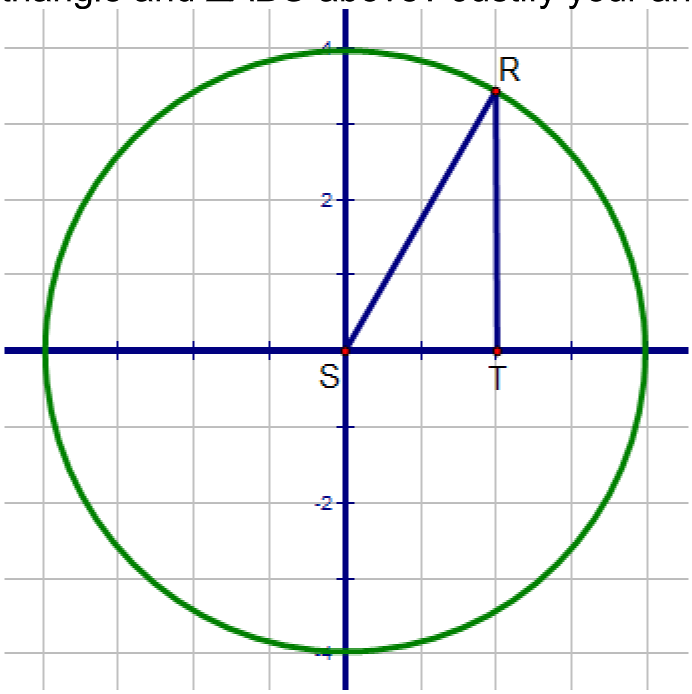
10. How can the coordinates at point *A* be used to find the values of the three trigonometric ratios? Explain your answer.

11. The blue triangle and green circle are the same size as the ones drawn above. The red circle and triangle *DBE* have been added. What can you tell about Δ*DBE*?




12. Find the following using Δ*DBE*.
    a. sin 30º = ___________
    b. cos 30º = ___________
    c. tan 30º = _____________

13. What do you know about $\Delta RST$? Hint: Is there a relationship between this triangle and $\Delta ABC$ above? Justify your answers.




14. Write the trigonometric ratios for the following:
    a. sin 60º = ___________
    b. cos 60º = ___________
    c. tan 60º = ___________

Use the page you created on “Triangle on a Grid” activity to answer the following questions.

1. On the triangle on the grid page, draw a right triangle with an acute angle that measures less than 30º and the hypotenuse the same length as the equilateral on the grid. Calculate each of the trigonometric ratios for the smallest angle in the new triangle. Determine if each trigonometric ratio is greater than, less than or equal to the trigonometric ratios for the 30º angle. Justify your answers.

2. Using the new triangle from #1, compare the trigonometric ratios of the angle greater than 60º to the trigonometric ratios for the 60º angle. Determine if each trigonometric ratio is greater than, less than or equal to the trigonometric ratios for the 60º angle. Justify your answers.

# Closing the Loop: Knowing Right

- Given a 30-60-90 special right triangle and the length of the short leg, I can find the measure of the other two sides by …

- Given a 30-60-90 special right triangle and the length of the long leg, I can find the measure of the other leg and the hypotenuse by …

- I can use the coordinates on the grid paper in order to find trigonometric ratios by …